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SERIES OF ARGUMENTS ON PROCESSES OF CRITIQUES TO MATHEMATICAL PROBLEMS

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Abstract

This study was initially based on the researcher's interest in a case found in students as they responded mathematical problems provided in the form of critiques to a problem. The study aimed to explore the students' arguments to describe the students' thinking processes while they are giving critiques to a mathematical problem. The study was qualitative research in a case study involving one student as a subject of research. The finding showed that the students used 4 series of arguments as the main reason to give critiques to the given problem. The critiques were delivered due to several factors consisting of; (1) the students' inability to discover appropriate strategies to deal with the given problems; (2) the personal experiences kept in a *Long Term Memory*, and (3) the *fallacy* on reasoning.

Keywords: Arguments, Critiques, and Mathematical Problems

Mathematical problems are considerably important for the development of mathematical science. Posamentier & Krulik (2009) consider problems as challenging questions, which are rarely found and hardly solved immediately. A problem is frequently given to make students implement the process of problem solving to find the best solution using all of their abilities (Yetik, Akyuz, & Kexer, 2012). There are several research discussing about problem solving such as Muis, Psaradellis, Lajoie, Leo, & Chevrier (2015) and Zsoldos-marchis (2015). Muis et al. (2015) states that the states of high curiosity, comforts and confusion affect positively on the cognitive and metacognitive uses while solving the mathematical problems. Zsoldos-marchis (2015) points out that the children positive behaviors on mathematics will affect to good problem solving skills and willingness of solving problems. One of significant problem solving is to deal with geometric problems.

Geometry is a remarkable branch of science and has been widely discussed in the research for the last 10 years. Some researchers investigated the geometric problem solving (Aydogdu& Kesan, 2014; Dejarnette & González, 2016; Warli, 2009). Warli (2009) state that in solving the geometric problems, the reflective students were very careful on the implementation stage and they observed various aspects, so their answers were relatively few in number but correct. In addition, the impulsive students were less careful on the implementation stage, so they got many answers but most were incorrect. Aydogdu & Kesan (2014) studied the strategies of geometric problem solving had by the Mathematic candidate teachers for elementary schools. Aydogdu & Kesan (2014) concluded that the strategies used by the 20 candidate teachers are 100% the strategy of drawing pictures, 30% the strategy of guessing and testing intelligences, 40% the strategy of brainstorming, 100% the strategy of prior knowledge and 65% the strategy of simplifying problems. Different from Aydogdu & Kesan (2014) and Warli (2009), Dejarnette & González (2016) conducted thematic analysis on the students' discussion while solving real-life problems using Geometry. Dejarnette & González (2016) illustrated the thematic items appearing when the students discussed about the context of problems in relation to the students' knowledge on geometry. Unlike the three studies, Hwang, Su, Huang, & Dong (2009) developed an innovation of virtual manipulation in a 3D room to discover clues in solving the geometric problems. The result of study by Hwang showed that the purposes system was considered useful, and helped the students to comprehend the process of solving geometric problems, like using various solving strategies. The strategies of problem solving were implemented after the teacher gave a problem.

The mathematical problems are given by teachers to allow students responding the problems and solving them; however, many students respond differently to the mathematical problems. Not few experts have examined several points in relation to students' responses to the mathematical problems (Eck, 2002; Koichu, 2017; Wu & Adams, 2006). Eck (2002) states that a response to a problem is actualized into processes to change or to solve the problem.Koichu (2017) identified factors stimulating or inhibiting the students' esthetical responses to figure out the problem. He points out that it is only mathematical problems with particular intrinsic characteristics which can be used as stimulation to trigger the esthetical responses in the problem solving. Wu & Adams (2006) studied students' responses from tests of problem solving which had been developed in the past. She analyzed the inhibiting factors in the problem solving to enhance students' abilities of problem solving.

In accordance with Eck (2002) arguments stating that a response to a problem is actualized into processes to change or to solve the problem, a difficult question was given to 89 junior high school students to identify their responses. The result showed that 88 students responded to answer all the given problems. There was 1 student responded the problem by criticizing that the problem was inaccurate due to the lack of information. Regarding with the phenomenon, it is apparently that all people will respond a problem to solve because of the absence of confidence to the given problems. The following is facts obtained from the students' responses to a problem.

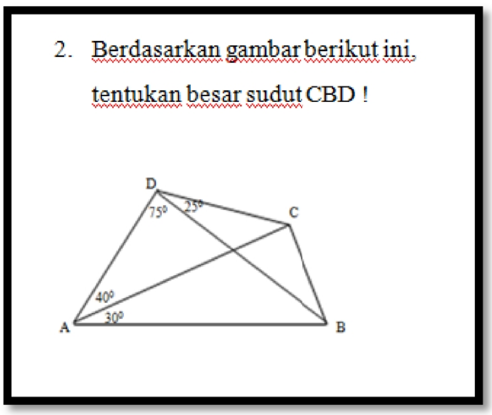


Figure 1a. The given problems

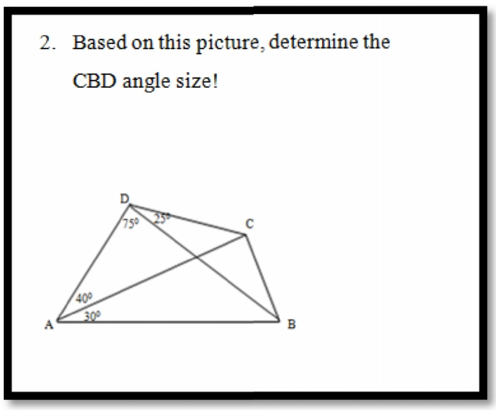


Figure 1b. Translation of the given problem

Students were instructed to solve problems. The students evidently criticize the problem based on several reasons, as follows by Figure 2.

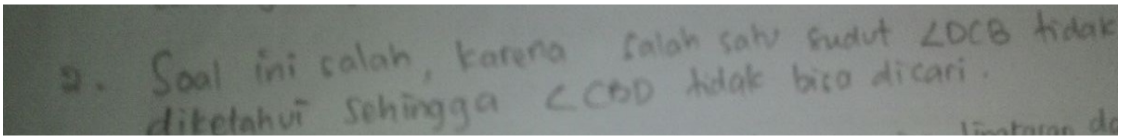


Figure 2. The students' written answers

> This question is false because one of the large of angles is unknown,
> the large of DCB angle is uknown, so the angle of CBD cannot be gotten.

Figure 2b. Translation of Figure 2a

In relation to the answers, it seems that students gave an argument to support their critiques. Arguments are results of a person reasoning. According to Eemeren, Houtlosser, & Henkemans, (2002) argumentation is considered as a type of communication purposed to solve different opinions. Good arguments are rational and have strong fundamental theories, so that the better a person's argument the more qualified his reasoning is. Therefore, it is required to analyze the arguments of students who criticize a problem. Critiques in this article are defined as an assessment, either good or bad, to something observed. This article aimed at analyzing students' arguments to describe students' thinking processes to a mathematical problem.

METHOD

This study was qualitative research conducting a case study purposing to explore students' arguments to describe thinking processes of students who give critiques to a mathematical problem (the geometric problem). The subject of study consisted of 1 student who criticized the mathematical problem. Next, the student was interviewed based on semi-structural interview guidelines to allow the student express the complete arguments in criticizing a mathematical problem. In the following, the arguments were analyzed to identify the students' thinking process while deciding to state that the mathematical problem was incorrect.

RESULT AND DISCUSSION

In accordance with the result of the given mathematical problem, the student responded as shown at Figure 2. The Figure 2 shows that the student used arguments with 3 propositions and 1 proposition (this problem was incorrect) which resulted in another 1 preposition (angle CBD was not known) and the preposition (angle CBD was not known) was a result from the other preposition (angle DCB was not known).Based on the result of interview, some information is gained as follows:

P: How can the angle DBC not be known? Is the question wrong?

S: A triangle should have angles 180° and it is known only 65°. So, it cannot be answered for sure [while pointing to the angle 65° the student wrote)

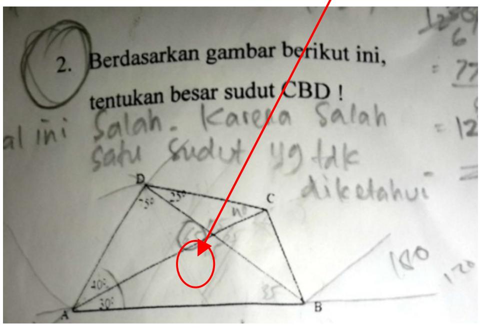


Figure 3. Written description in the interview

* Note : Translated version of the reason is "this question is false because one of the amount of angles is unknown"

It is shown that the student acquires knowledge that the triangle angles are 180 degrees and since there are 3 angle areas in a triangle, so the student concluded that it is impossible to know the degree of one of the angle areas if the initial assumption is only known the degree of one angle area only. After the profound interview, it was obtained 4 series of arguments given by the student in summing up that the degree of angle "cred circled" is 65°. The details are described in the following table.

Table 1. Series of the First Argument

Result	Reason	The used knowledge
65°	The vertical angle is 65°	A vertical angel is congruent
65°	The adjacent angle is 115°	A pair of adjacent angles is the sum of two angles equals to 180°
115°	Two other angles are 40°and 25°	The sum of angles in a triangle is 180°
25°	It is known in the problem	
40°	Two other angles are 100° and 40°	The sum of angles in a triangle is 180°
100°	The result of sum from two known angles, 25°and 75°	
40°	It is known in the problem	

Table 2. Series of the Second Argument

Result	Reason	The used knowledge
65°	The adjacent angle is 115°	A pair of adjacent angles is the sum of two angles equals to 180°
115°	Two other angles are 35° and 30°	The sum of angles in a triangle is 180°
35°	Two other angles are 75° and 70°	The sum of angles in a triangle is 180°
75°	It is known in the problem	
70°	The sum oftwo known angles, 40° and 30°	

Table 3. Series of the Third Argument

Result	Reason	The used knowledge
65°	The vertical angle is 65°	Vertical angles are congruent
65°	Two other angles are 75° and 40°	The sum of angles in a triangle is 180°
75°	It is known in the problem	
40°	It is known in the problem	

Table 4. Series of the Fourth Argument

Result	Reason	The used knowledge
65°	The adjacent angle is 115°	Adjacent angles arethe sum of two angles equals to 180°
115°	Two other angles are 25° and 40°	The sum of angles in a triangle is 180°
25°	It is known in the problem	
40°	Two other angles are 100° and 40°	The sum of angles in a triangle is 180°
100°	The result of sum from two known angles, 25° and 75°	
40°	It is known in the problem	

According to the 4 series of arguments, a statement is shown that the student cannot determine the angle questioned in the mathematical problem. It means that after the result of 65° was found, the student cannot proceed or develop the 65° to solve the given problem. Based on the 4 series of arguments, it seems that the student cannot figure out the mathematical problem despite using 4 strategies of problem solving. In accordance with the students desperation and retention concerning the experience in the elementary school (the student could not solve the problem because the problem was eventually incorrect), the student concluded that the recent problem faced was also wrong. The general schemes of the student thinking process in relation to the arguments given by the student are presented as follows by Figure 4.

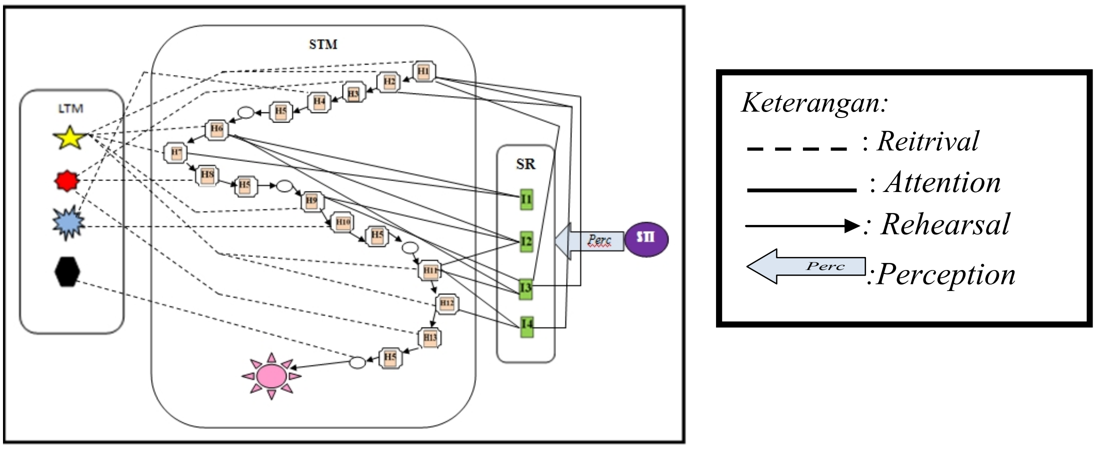
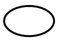
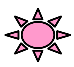


Figure 4. The student's Thinking Scheme to Criticize a Mathematical Problem

Table 5. The Meaning of Codes and Symbols in the Student's Thinking Scheme

Codes (symbols)	Meaning
	The sum of angles in a triangle is 180^0
	Adjacent angles are the sum of two angles equals to 180°
	Vertical angles are congruent
	The problem which cannot be solved is usually a wrong problem
	The process of information which becomes attention is based on memory called as Long Term Memory
I1	Angle BAO = 30^0
I2	AngleDAO = 40^0
I3	AngleADB = 75^0
I4	AngleBDC = 25^0
H1	Angle ACB = 40^0 because $180^0 - 40^0 - 75^0 - 25^0 = 40^0$
H2	AngleCOD = 115^0because $180^0 - 40^0 - 25^0 = 115^0$
H3	AngleAOD = 65^0 becasue $180^0 - 115^0 = 65^0$
H4	AngleBOC = 65^0 becauseangle BOC is corresponding to AOD
H5	The questioned angle could not be known because it is only angle BOC known in the triangle.
H6	Angle ABO= 35^0 because $180^0 - 40^0 - 75^0 - 30^0 = 35^0$
H7	AngleAOB = 115^0 because $180^0 - 30^0 - 35^0 = 115^0$
H8	AngleBOC = 65^0 because $180^0 - 115^0 = 65^0$
H9	AngleAOD = 65^0 because $180^0 - 40^0 - 75^0 = 65^0$
H10	AngleBOC = 65^0 because angel BOC is corresponding to AOD
H11	Angle ACB = 40^0 because $180^0 - 40^0 - 75^0 - 25^0 = 40^0$
H12	AngleCOD = 115^0 because $180^0 - 40^0 - 25^0 = 115^0$
H13	AngleBOC = 65^0 because $180^0 - 115^0 = 65^0$
	The student did reflection as the given strategies could not solve the given problem.
	The student criticized the given problem by providing astatement that the problem was incorrect.

Based on the finding, it is shown that 4 series of the student's arguments initially appeared from the presence of reflection to the first series of arguments (the series of arguments showed a strategy used to solve problems) implemented by the student due to the students' inability to proceed the used strategies further to achieve a solution of the given problem. The student's reflection to the first series of strategies was shown by the decision to use new strategies in solving problems. The new strategy was the second strategy. The decision to implement the new strategy was based on the inability to find the solution.

The emergence of the third and the fourth strategies was due to the presence of reflection to the previous strategies which was unable to solve problems. The fourth strategy used was a form of fallacy in reasoning started by the use of wrong premises so it resulted in a wrong conclusion. The fourth strategy showed the same result and was not able to figure out the given mathematical problem.

The inability to solve the problem was a result from the fallacy of reasoning; consequently, it resulted in a decision (Dorr, 2016). For this reason, the fourth strategy became the student's strongest argument to point out that the given problem was incorrect. The student's decided to state that the problem was wrong because from the four implemented strategies were not found the bright side to get the solution and the student's *Long Term Memory* saves the experience that the problem of National Examination Try Out which the student could not solve was a problem which the teacher mentioned it as a wrong problem. It is shown that the student's decision to criticize the problem based on the priorexperience saved in *Long Term Memory.*This is similar to Pawestri, Wardani, & Sonna (2013) argument that an individual attitude is influenced by the acquired knowledge and awareness to act.

CONCLUSION

In accordance with the finding and discussion, it can be concluded that students implemented 4 strategies of arguments as the main reason to criticize the given problem.The overall identification of the factors causing the critiques to the given problem is; (1) the student's inability to discover the appropriate strategy to solve the given problem; (2) the personal experience saved in *Long Term Memory* and (3) the *fallacy* in reasoning.

According to the study conducted by the researcher, there are several unanswered questions. The questions are recommended to be investigated in the following studies. The questions are: (1)How the students' critical thinking process to solve problems which include at least two contrary information. (2) Howthe process of the emergence of *Attention* to information in mathematical problems received from *Sensory Register.*

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