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THE EMERGENCE OF STUDENTS' METACOGNITION IN THE PROCESS OF MATHEMATICAL PROBLEM SOLVING

Nopem Kusumaningtyas[1], Cholis Sa'dijah[2], Rustanto Rahardi[3], Swasono Raharjo[4]

[1]IKIP Budi Utomo, Malang, Indonesia
[2,3,4]Universitas Negeri Malang, Indonesia
nopem_kusumaningtyas@yahoo.co.id

Abstract

Developing a problem-solving skill for important things which need to be paid attention is how the students' thinking process in solving a problem is. Thinking about what is thought in this term relates to students' awareness of their ability to develop various ways which may be taken in solving a problem is known as metacognition. The existence of metacognition is very important for students in solving mathematical problems. This is a qualitative study using the grade VII students having medium ability as the subjects. The results of the study obtained that through intervention conducted by the researchers in 5 meetings, metacognition capability emerged in the subjects so that with their own ability they are able to solve mathematical problems provided.

Keywords: Metacognition; Problem Solving

Every day in our life we must often be faced by problems since problem and problem solving process are parts of the self-maturation process which must be passed and the self-existence of as an individual or as a part of environment. Thus, the ability to solve a problem is a skill which must be owned by someone to be able to take a better life. The discussion in this study is not intended for the whole problems, but rather focuses on problems related to mathematics lesson at school.

Problems in mathematics lesson at school is a routine and non-routine problem and related with a given concept and related to mathematics. If a student in the class can solve mathematical problems given during the learning process, the formulation of learning objectives which have been formulated are achieved. Therefore, the ability of students to solve mathematical problems needs to be trained so that students are able to solve existing problems in mathematics.

Students need to have problem solving skills (NCTM, 2014; Posamentier & Krulik, 2009). Posamentier & Krulik, 2009say that mathematical problem solving is a basic skill in learning mathematics. Mathematical problem solving is a general goal of mathematics learning, problem solving includes methods, procedures, and strategies are the core and major process in the mathematics curriculum. NCTM (2014) says that problem-solving ability is one of mathematical abilities which students must possess.

To assist students in developing problem-solving skills, the important thing which needs to get attention is how the students' thinking process in solving a problem is. Desoete, Roeyers, & Buysse(1996)statement that the main purpose of teaching problem-solving in mathematics is not only to equip students with a set of skills or processes but rather to allow students to think about what they are thinking.

Thinking about what is thought in this terms relates to the student's awareness of their ability to develop various ways which may be taken in solving problems. According to Mokos & Kafoussi, (2013), the process of realizing and regulating students' own thinking is known as metacognition.

Metacognition is defined as thinking about what someone is thinking. In general, according to Flavell (1979) metacognition deals with two-dimensional thinking, namely (1) *self-awareness of cognition,* the knowledge owned by someone about his/her own thinking, and (2) *self-regulation of cognition,* the someone's ability to use his/her awareness to regulate his/her own cognitive processes. In relation to problem solving this is consistent with Brown's view which divides metacognition into two categories: (1) knowledge of cognition, as an activity which includes conscious reflection on someone's thinking and activity, (2) cognition arrangement, as an activity paying attention to the self-regulatory during the ongoing of effort to learn or solve problems (Ozsoy & Ataman, 2009).

In relation with the problem-solving process Mokos & Kafoussi, (2013) states that metacognition is known as a key factor in solving a problem, including: (1) determine the knowledge owned; (2) formulate the plan for resolution; (3) choose the solving strategy; and (4) monitor and evaluate the activities used during the problem-solving. Thus, metacognition can assist students in solving problems ranging from exploring the knowledge they have to solve problems, arranging the solution plan, monitoring the thinking processes in the problem solving, and evaluating the process and problem-solving outcomes. It can be said that the existence of metacognitive is very important to be owned by students in solving mathematical problems.

Therefore the existence of metacognition in the problem solving becomes one of the interesting factors which has so much attention by educational researchers such as studies conducted by Wong (2007), Kuzle (2013), Mokos & Kafoussi(2013), Karan & Irizarry(2014), and In'am, Saad, & Ghani(2012). These are due to the benefits which can be gained when problem-solving has done by involving awareness to the thinking process and self-regulatory ability thus these allow a strong and comprehensive understanding of the problem with logical reasons. Based on the description above, this study was conducted to contribute in the development of knowledge focusing on how to create metacognition in the process of mathematical problem solving was.

METHOD

This study used deskriptifeksploratiftype. The subjects in this study were one medium-skilled student selected from grade VII students from one public school in Sidoarjo Regency. The medium ability is gained from the results of problem solving done by the student before this study done. The study was conducted for 7 meetings. In 1st and 7th meeting, the researchers did not provide any intervention on the subject when the problem was processed, whereas for the 2-6th meeting the researcher intervened on the subject according to the problems found on the subject while solving the mathematical problem given. Intervention done by the researchers more on showing how the steps to be done in problem-solving, showing skills which can be used in solving mathematical problems. Each meeting had different times depending on the researchers' intervention, what was done, and the problem found on the subject in the problem-solving process.

There were 4 different mathematical problems given by the researcher in this study. The 1st problem was given in the 1st-3rdmeeting, the 2nd problem was given in the 4th-5th meeting, while the 6th, 7th meeting each worked on 3rd and 4th problem. The problems given are appropriate to the material being studied by the subject at the school.

In addition, this study also used interview process and *thinking aloud* method to know what the student was thinking during solving the mathematical problems. According to Ericsson and Simon (Mokos & Kafoussi, 2013) while using *thinking aloud* method subjects will report every thought they make. In this study the subject should explain out loud why he was taking it, consider how it was done, or how it solved the problems.

Data collection was done by recording or documenting the process done by the subject in problem solving and interview was conducted by the researchers on the subject. The results were obtained from the data collection analyzed, then the results of the analysis were presented in the form of narrative text.

RESULT AND DISCUSSION

This study was conducted for 7 meetings, the following results got from each meeting;.

First Meeting

In the first meeting the subject did the mathematical problem given by the researchers. The researchers assigned a mathematical problem to the subject at the first meeting. The problem given by the researchers as follows:

> "You are going to buy drinks for your party in various packages: for 500 ml its cost is Rp 2,500,00; 1liter is Rp 4,250,00; 1.5 liter is Rp 6,000.00. You know that 1liter = 1,000ml. In the party you requires drinks as much as 6liters. Which package do you think is the cheapest for the party you are going to have?"

The problem given by the researchers in this first meeting is a type of contextual problem adopted from a mathematical problem given by Mokos & Kafoussi(2013)on his study.The following figure is the result of the subject's work in the first meeting;

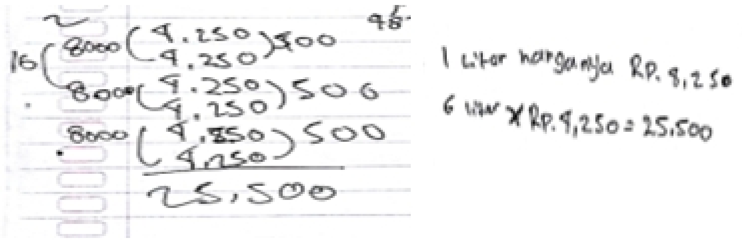

Figure 1.

The subject had not understood what was being asked in the problem. This is recognized in the reading process of the problem given, the subject read while scratching his head. By reading at once the subject directly interpreted that the problem's question was about which package is the cheapest of the three packages and how much it should be paid for 6 liters. After reading, the subject immediately did as he thought about what is being asked in the problem.

The steps taken by the subject were in accordance with his assumption about what is being asked in the problem. Then he wrote 1 liter is 4,250 on his worksheet but in counting 4,250 for 6 times, he did not multiply 6 by 4,250. In his draft he wrote 4,250 as much as 6 in layers and then he summed them up.The subject had not had the correct answer yet for the mathematical problem given. After doing the mathematical problem, the subject did not check his work again.

In the 2nd-6th meeting the researchers gave intervention on the subject in each meeting. The interventions are tailored to the problems found when the subject was working on the mathematical problems. Therefore, the intervention done by the researchers in every meeting varied.

Second Meeting

In the second meeting, the researchers asked the subject to do the mathematical problem in the first meeting again. But before doing the work, the researcher gave an intervention on the subject about the steps in the problem solving. The intervention given by the researcher are seen in the following conversation:

Researcher	:	*If you are working on a problem, read it over and over again until you understand the purpose and know what is being asked in the question.*
Subject	:	*Yes, Ma'am.*
Researcher	:	*OK, now try to read the problem I gave yesterday.*
Subject	:	*(Read aloud the problem given in the first meeting at 2 times)* *Ma'am, I've already understood the purpose of this problem.*
Researcher	:	*What do you understand about the problem?*
Subject	:	*There are 3 packages namely 0.5liter, 1liter, and 1.5liter. Each package has different price. Well, if the party to be done takes 6liters, which package is the cheapest? That's what this problem's purpose, isn't it, Ma'am?*
Researcher	:	*That's right. Good job. Now, after you know what this problem's purpose is, what will you do?*

Subject : *Um ... (while looking at the question sheet).*

Researcher : *Um ... what?*

Subject : *Please, wait, Ma'am, I'm still thinking.*

Researcher : *What are you thinking about?*

Subject : *(Write 0.5 + 0.5 = 1, 1 + 1 + 1 + 1 + 1 + 1 = 6, 6 x 2)*
For 0.5 liter package it means I take 12 bottles. 1liter takes 6 bottles. (Write again 1.5 + 1.5 = 3, 3 + 3 = 6, 2 x 2 = 4)
1.5liter takes 4 bottles.

Researcher : *Then ...*

Subject : *Then, 12 bottles are multiplied by 2,500, 6 bottles are multiplied by 4,250, and 4 bottles are multiplied by 6,000.*
(for 12 bottles and 6 bottles the calculations are arranged down then summed, whereas for bottles it is directly 4 x 6,000)
12 bottles need the money of Rp30,000; 6 bottles need the money of Rp25,500; and 4 bottles need the money of Rp24,000. It means that I will spend the fewest money if buy 4 bottles, 4 bottles of 1.5liters package.

Researcher : *Good. We continue our meeting next week, OK.*

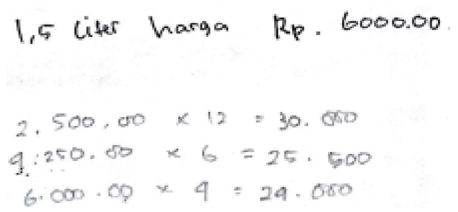

Figure 2

In that meeting the researchers gave an intervention by asking the subject to repeat reading the problem given until he understands the purpose and knows what is being asked. This was done by the researchers based on the first meeting, the subject had not understood the problem and directly worked on it. As a result the subject's answer was not correct for the mathematical problem given.

Third Meeting

In the third meeting the researchers still used the same mathematical problem. The researchers continue to intervene the subject in this meeting. The conversation among the researchers and the subject in the third meeting as follows.

Researcher : *We continue the yesterday's meeting, OK.*

Subject : *Yes, Ma'am.*

Researcher : *Now look at the result you did yesterday, the time you need was very long to solve the problem. Well, in solving a problem you have to estimate the time required in doing it. You have to think about ways to work on the problem, then, choose the fastest way in solving. When choosing a way of solving, use*

		knowledge you have which can be used to solve the problem you are doing. Now let's see if there's any other way around the problem you did in the yesterday's meeting.
Subject	"	*Um ... please wait, Ma'am (while pointing at the answer he did yesaterday). If I do like this way, what do you think, Ma'am? (writing while talking).*
		12, 6, and 4 bottles I get that 6liters are divided by the package size. In order to calculate it easily, I change liters into milliliters. Like this way, Ma'am, if I use this way, it is easier and faster to calculate than the way I used yesterday.
Researcher	:	*Is there another way?*
Subject	:	*It seems like there's no another way, Ma'am.*
Researcher	:	*Now try to recheck your answer, is it true of how you did in solving the problem?*
Subject	:	*(Looking back at the answer) It's true, Ma'am.*
Researcher	:	*OK. Our meeting today is over. Reflect and apply the way I gave to you when you work on problems in the future. See you tomorrow.*
Subject	:	*Thank you and see you, Ma'am.*

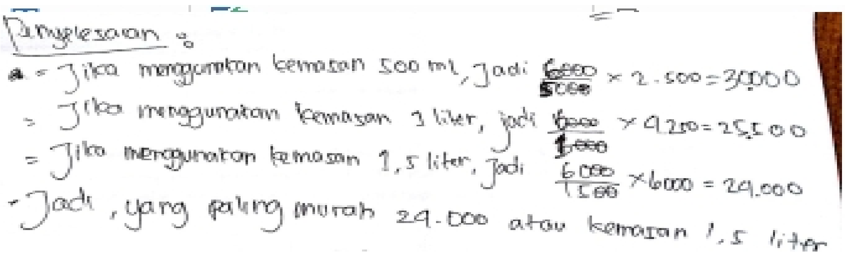

Figure 3.

In the third meeting the researchers intervened by asking student to consider the time needed to solve the mathematical problem. This is based on the previous meeting where the subject took a long time to work on the problem given. In addition, the researchers asked the subject to use his prior knowledge in searching for ways of doing problems. Finally, the researchers asked the subject to check the answer of the results of his work.

Fourth and Fifth Meeting

In the fourth meeting the researchers took different problems from the 1st meeting. The problem given by the researchers were as follows:

> "A person buys a house with a size of $90m^2$ which its price is US$150,000.00/meter. He pays the down payment of the house. The remaining price is paid in installments for 25 months. How much the installment should he pay each month?"

The fourth and fifth meetings provide intervention on the subject in solving the mathematical problem given.

Before the subject would work on the problem, the researchers provided an intervention. The interventions conducted by the researchers are as follows.

1. The researchers asked the subject to read repeatedly to understand the problem given.
2. The researchers asked the subject to write down what is known and asked in the draft paper.
3. When determining the way of problem solving the subject was asked to relate it to the knowledge he has with the problem given.

At the time of the process of problem-solving the researcher reminded the subject to look at the time in the process of problem-solving and think about what strategy used to make in order to solve the problem precisely and in fast. In addition, the researchers also gave help when the subject encountered a difficulty in the process of problem-solving. The difficulty which the subject had was when he calculated the house price. When he calculated it, the subject wrote down all the zeroes in the house price, finally a mistake in counting happened. Because of that mistake the researchers gave a help by suggesting "what if you remove all of the zeroes, just after the calculation finish, add the zeroes in accordance with the zero removed. The following figure is the subject's work.

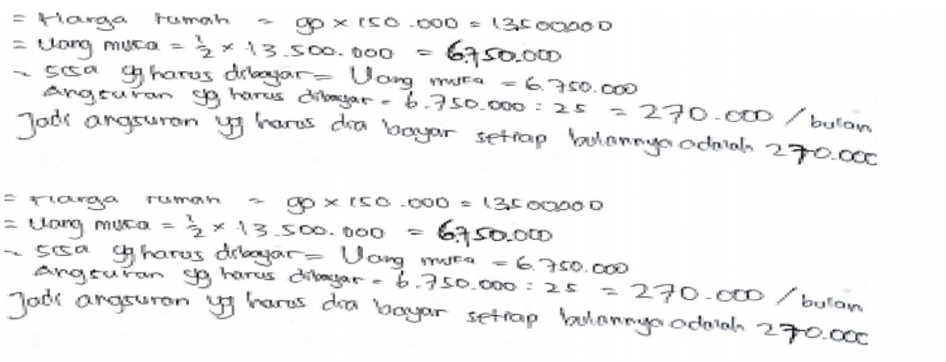
= Harga rumah = 90 × 150.000 = 13500000
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Figure 4.

After the subject solving the mathematical problem, the researcher asked the subject recheck his work. At the end of the meeting the researcher told to the subject about what the subject got during the five meetings can be applied in the mathematical problems in the future.

Sixth Meeting

In the sixth meeting, the mathematical material being studied by the subject was triangle. To adjust to the subject being studied the researcher gave a mathematical problem relating to triangle. This was done researchers since the knowledge about the material is still remembered in the memory. The mathematical problems is as follows.

Which are the possibilities to form triangle if there are long sticks provided in the following size? Investigate!

a. 11 cm, 12 cm, and 15 cm
b. 2 cm, 3 cm, and 6 cm
c. 6 cm, 10 cm, and 13 cm

d. 5 cm, 10 cm, and 15 cm

In this meeting, the researchers still intervened on the subject in working on the problem given. The following interventions conducted by researchers on the subject:

Researcher	:	*I have a question about triangle, try to solve the problem.*
Subject	:	*Yes, Ma'am. (Reading the question given by the researchers)* *From this problem it is known there are 4 groups of long sticks. Each group has 3 different lengths. This question asks which of the four groups can form a triangle.*
Researcher	:	*Nice. And then ...*
Subject	:	*The four groups are all triangle, Ma'am.*
Researcher	:	*Why is all triangle?*
Subject	:	*Because the four groups have three sticks, there are short sticks and there are long sticks. Triangle has tree sides and from the three sides there are short and long sticks, am I right, Ma'am?*
Researcher	:	*Are you sure with your answer?*
Subject	:	*Um ... please, wait, Ma'am. (While drawing 2 right triangles of triple Pythagoras, 3, 4, 5 and 6, 8, 10).* *3+4 is bigger than 5, 6+8 is also bigger than 10. It means the above problem is also like that. (Directly writing it on the answer sheet)* *Evidently, the groups which can form triangle are groups a and c, Ma'am, because the short sides of group b if they are summed are smaller than the long side, and for group d if I sum the short sides, the result is equal to the long side.*
Researcher	:	*Are you sure with your answer?*
Subject	:	*Let me check it again, Ma'am. (Reading the problem again after correcting the answer) I'm really sure that it's right, Ma'am.*
Researcher	:	*OK. Thanks a lot. We'll meet again on Tuesday. Will you come?*
Subject	:	*Yes, I will, Ma'am. I also say thank you very much, Ma'am.*

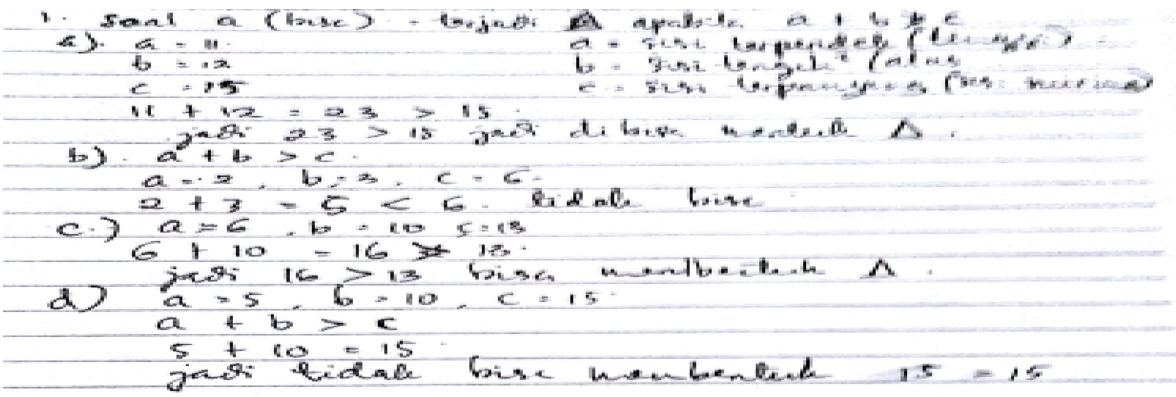

Figure 5

In this sixth meeting, the researcher intervened a bit with the question of '*are you sure with your answ*er' so that it emerged awareness, evaluation, and self-regulation of metacognitive in the subject while solving the mathematical problem.

Seventh Meeting

In the seventh meeting the subject was given a different problem again. The problem given was still related to the triangle material because in the school the subject is still discussing about the triangle. In the seventh meeting the researcher did not give any further intervention. The subject worked on problem-solving by using *thinking aloud* method.

The mathematical problem given by the researchers is as follows.

> "If you get four triangles (arbitrary triangle, right triangle, equilateral triangle, and isosceles triangle) which have the same circumference of 24 cm, determine which type of triangle having the largest area!"

The results of the subject's work in solving the mathematical problem is showed in Figure 6.

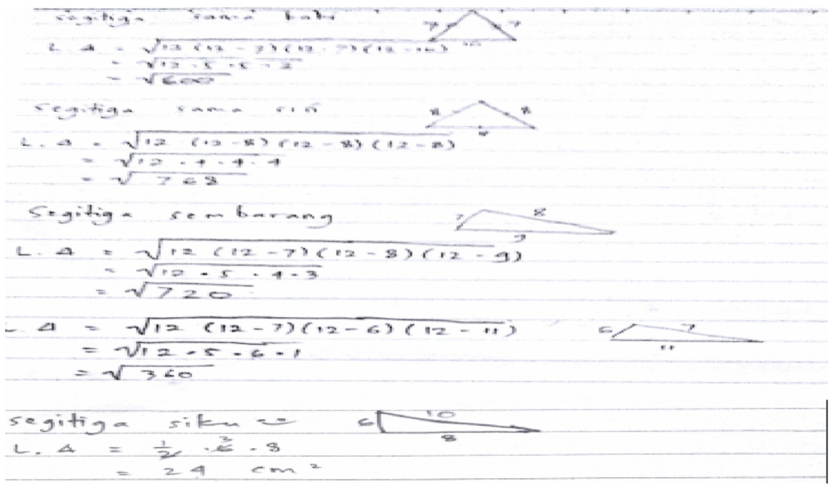

Figure 6

Without any intervention by the researchers the subject's metacognition emerged in the process of problem-solving. From his '*thinking aloud'* the subject's metacognition was seen in the problem-solving process as follows.

1. When the subject would determine the length of the sides of isosceles triangle and arbitrary triangle, he said, "If I choose a decimal number for the length of the sides, it'll come later difficulties when calculating the area, meaning that I have to choose a round number for the length of the sides". Finally, he chose the length of the sides of 6, 7, 11 for the arbitrary triangle and 7, 7, 10 for the isosceles triangle.

2. After determining the lengths of the four triangles the subject's metacognition reemerged when he would calculate the area of the four triangles. Before calculating the areas he writes 2 equations of the area of triangle namely:

   a. L = $\frac{1}{2} \times a \times t$

   b. L = $\sqrt{s\,(s-a)(s-b)(s-c)}$

   "If I choose the first width formula, it is easy for me to calculate the area of a right triangle, but, it will be difficult and takes a long time to calculate the area of 3 other triangles because I have to look for heights first. It seems like I'll use the second formula, the circumference of which is known, so to calculate the area I just put the circumference and the lengths of the sides of each triangle.

In short, besides metacognition emerges in the process of solving mathematical problem, the subject also does what was given by the researchers at previous meetings, for instance, repeating the reading of the problem given until he understood his problem, taking the time to define the strategy of solving, linking the knowledge he has before, and re-checking with what he did.

In this study, the researchers performed 7 meetings. For the 1st meeting, the researchers did not give any intervention to the subject while solving the problem, with the purpose of knowing how the process undertaken by the subject in analyzing the problem given. From the results of the study it shows that the subject could not solve the problem given. Then in the 2nd-6th meeting the researchers intervened the process of problem solving conducted the by subject. The purpose of this intervention was to allow the subject to know what to do when solving a mathematical problem. Interventions given are in accordance with the problems found in each meeting. These interventions decreased with the increasing number of meeting, until in the 7th meeting the researchers did not give any intervention on the subject while solving the mathematical problem given.

From the results of the study it shows that after interventions done by the researchers in the 2nd-6th meeting, the subjects who originally could not solve the mathematical problem, could solve the mathematical problem in the 7th meeting. In addition, the subject was not only able to solve the problem, the subject's metacognition also emerged in the process of problem solving of the mathematical problems given. With the emergence subject's metacognition, it certainly helps students to solve the problems given. This is consistent with the results of the studies done by Howard & Ellis(2005);(Marta T. Magiera (2011); Kuzle(2013); Karan & Irizarry(2014), and Mokos & Kafoussi(2013) who say that metacognition strategies have an effect on students' problem-solving ability.

CONCLUSION

Based on the results of the study it is found that through the gradual intervention for 5 meetings on the subjects, the metacognition emerged from the subjects on the problem-solving process. By the emergence of metacognition in the subject, this causes the subjects to be able to solve the mathematical problems given. For future studies, it is necessary to think about how the study is done by not on a subject but also classically. Therefore, it is necessary to develop a learning model which can emerge metacognition on learners so that it is not only to make learners able to solve problems but also able to improve learning outcomes.

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