ISSN 2615-2495

**International Journal of Insights for Mathematics Teaching**
Volume 02, No. 1, April 2019, pp. 22-32

# ELEMENTARY STUDENTS' INTUITIVE UNDERSTANDING TO CONSTRUCT THE AREA MEASUREMENT

Mohammad Faizal Amir

Universitas Muhammadiyah Sidoarjo, Indonesia
faizal.amir@umsida.ac.id

Students fail to understand the concept of the area because they do not have an intuitive. As revealed in a number of previous studies, this case occurred because of the absence of structured, intuitive understanding, at the early level of elementary school. The present research involving 20 third graders of elementary school focuses on strategies construction and intuitive development in the area measurement. In this descriptive qualitative study, the students are given three tasks at different times and based on the result of each task, a number of students are selected to be interviewed. The present study found two novel strategies in addition to (Lynne N & Michael C, 2000) proposed ideas, i.e., visual-concrete and measurement estimation. In line with this, there are two additional levels of transformation intuitive understanding development. In this case, the researcher names them as level 2 and level 4. Therefore, the levels now can be formulated as level 0: incomplete cover, level 1: primitive cover, level 2: visual-concrete cover, level 3: covering arrangement built of units, level 4: cover built with estimation measurements, level 5: array built by measurement, level 6: implied array, calculation solution. This finding has implications for the development of an intuitive understanding that will be more detailed. Thus, a teacher can make a teaching and learning plan strategy more adequate to construct the area measurement for early elementary school students.

**Keywords: intuitive understanding; strategies; intuitive development; area**

Understanding is a very fundamental element in the purpose of learning mathematics. Almost all studies in mathematics education make understanding the focus of discussion, including assessment (Berenson & Carter, 1995); curriculum development (NCTM, 1989); problem solving (Baranes, Perry, & Stigler, 1989); teaching and learning (Ausubel, 1968; J. Hiebert & Carpenter, 1992; Linchevski & Kutscher, 1998). It is different from understanding (Skemp, 1987) which has been known so far, namely relational understanding, "knowing both what to do and why" and instrumental understanding of "rules without reasons." Some experts (Byers & Herscovics, 1977; Pirie, 1988) propose intuition on aspects of understanding which are generally equated with "guessing" so that the steps seem to be unstructured, not well defined, and include little awareness of the process being carried out, also have difficulty explaining.

In relation to one of the most commonly used measurement domains in everyday life and is the basis of concepts to explain integer multiplication (Hirstein, Lamb, & Osborne, 1978; NCTM, 2010), broad models also become natural means of teaching fractions and multiplication (Freudental, 1983). The intuitive understanding of broad measurements lies in the approach in the form of actions to cover the surface of a large area, both using a particular building unit in the form of concrete or not systematically, before reaching the formal stage, where students can cover the area appropriately and realize the formula of the area formed (Lynne N & Michael C, 2000; Outhred & Mitchelmore, 2004).

Research related to an intuitive understanding of elementary school students has been conducted by (Lynne N & Michael C, 2000) in constructing intuitive strategies used by early grade elementary school students to measure area. The results of this study find the intuitive strategies students include incomplete covering, visual covering, inadequate array, concrete covering, array estimation, array draw, measurement (the measurement of one dimension/measurement of both dimensions), array draw implied, and array calculated. These strategies are classified into five stages of development, which are level 0: incomplete covering, level 1: primitive covering, level 2: array covering constructed from units, level 3: covering constructed by measurement arrays, level 4: array implied, solution by calculation. One of the tasks given aims to make students determine the number of square units in the form of concrete objects measuring 2 cm that can cover an 8cm square. Figure 1 shows the strategy of student answers consisting of incomplete covering, visual covering, concrete covering, and measurement.

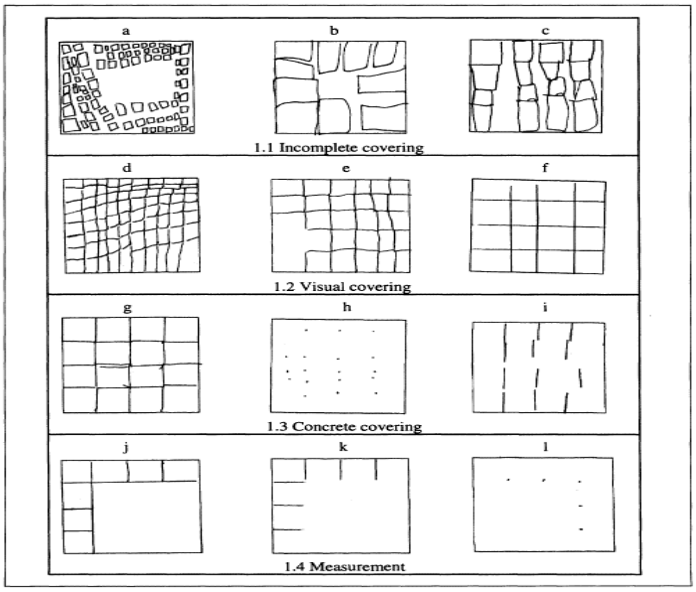


Figure. 1. Students Intuitive Strategy Results in Task No.1

To construct intuitive developmental stages, (Lynne N & Michael C, 2000) laid the basis of the emerging empirical evidence of the students that appeared. That is, for example, if the area covered is not square or rectangular, and does not use a unit square. The strategy and development of intuitive understanding that appears will likely be different. Indeed, although the research carried out is about measuring the area, unit squares are often used to cover a square or rectangular area (Battista, Clements, Arnoff, Battista, & Borrow, 1998; Hirstein et al., 1978; Outhred & Mitchelmore, 2004). But this is not necessary because the use of other than square units to cover a certain flat building is still possible. The most important thing in measuring the area can integrate units into spatial and numerical concepts (James Hiebert, 1981).

The parallelogram has a shape that is different from a square or rectangle when partitioned perpendicular to its base from the opposite vertex. The parallelogram consists of triangles and squares or rectangles (Bennett, Burton, & Nelson, 2012). This research reports the results of empirical studies in the form of intuitive strategies, elementary school students use more varied square and unit triangles to cover the parallelogram so that it impacts at a more detailed stage of the development of understanding. These findings provide very important implications in mathematics education, especially in terms of broad measurements, which will show the right strategies to teach broad measures. This will also show the recommendations for the sequence of how the teacher teaches correctly so that there is no cognitive leap. Also, the teacher will find out how they should teach so that it is in line with the development of the understanding of the students' motivation.

## METHOD

This research is a qualitative descriptive one. The subjects in this study were 20 3rd grade students of the elementary school in Sidoarjo, Indonesia. To investigate intuitive comprehension strategies, students are given Task 1, Task 2, and Task 3. The results of the answers to the subjects are grouped based on the characteristics of the strategy of the spatial structure that appears to be interviewed randomly on each subject representing the characteristics of the strategy (Creswell, 2012). Task 1 includes how many unit squares and triangles are needed to cover the area in the square, if the parallelogram is 10cm in length, 8cm in height, while the square unit is 2cm x 2cm, triangle 1 has a height of 2cm and a base of 1cm, triangle 2 height 2cm and base 1/2 cm. However, parallelogram and unit sizes are not indicated. This is done to avoid obstacles in the thinking process of students in constructing the purpose of the task and to keep students from tending a measurement strategy. Thus, the strategy and development of students' understanding are still within the scope of intuitive understanding when completing Tasks.

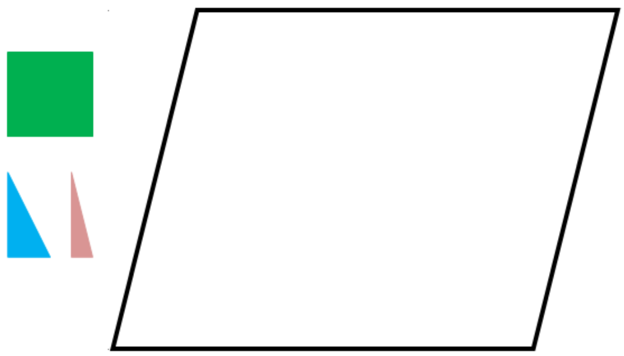

Figure 2. Parallelogram, triangle and square units in Task 1

Unlike Task 1, concrete objects are not given to Task 2 but are only given a visual form, and the parallelogram sizes and units are still not the same. Before working on Task 2, students are given the pre-task of measuring the lines as Figure 3.

Figure 3. A line with certain length as pre-Task 1

Students are given the task of determining how many square objects and unit triangles are used to cover the parallelogram, as shown in Figure 4.

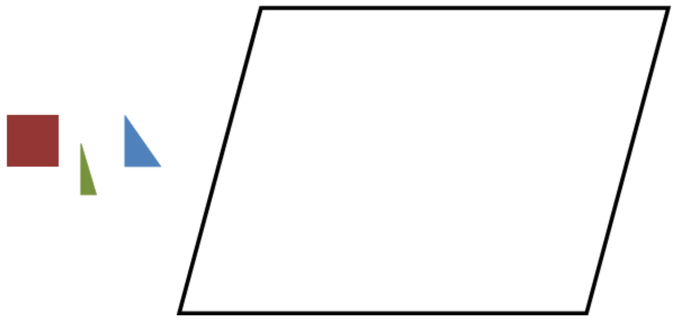

Figure 4. Parallelogram, triangle and square units in Task 2

Unlike Task 1 and Task 2, the students are not given concrete objects or visual forms to represent the unit given in Task 3, but the unit and length measurements are presented. If there is a unit square with a size of 2cm x 2cm and a triangle that has a size of ½ of a unit square, then what is the unit square and triangle used to cover that part if the length is 8cm x 10cm?.

## RESULT AND DISCUSSION

As an initial step before analyzing the stages of development of an intuitive understanding of students, the results of task 1 to task 3 are grouped according to empirical strategy data of students based on the category of strategies according to (Lynne N & Michael C, 2000).

***Intuitive Task 1 Strategy***

Characteristics Intuitive strategies that appear in the results of task 1 are mostly influenced by concrete square or unit triangles to cover the parallelogram area. Figure 5 shows the variation of strategies carried out by students.

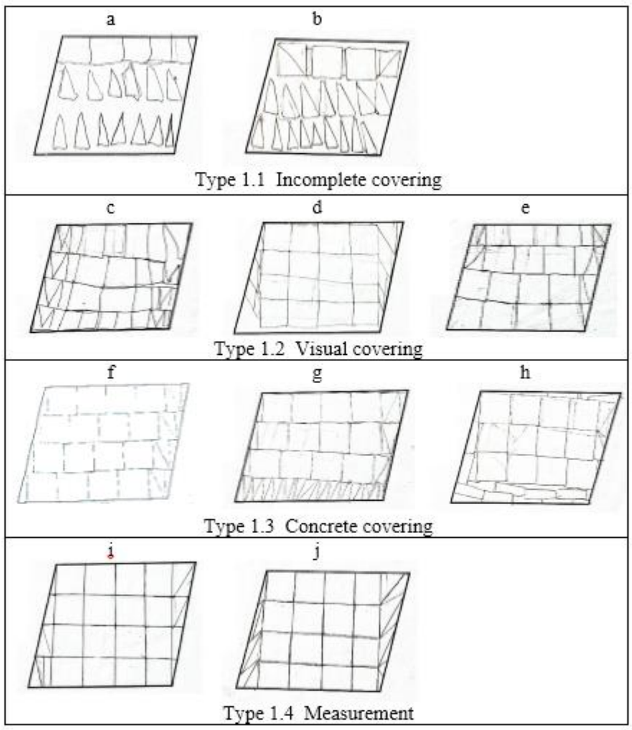


Figure 5. Result of Intuitive Strategy on Task 1

*Incomplete covering Strategy*

Strategies in this type have characteristics of students failing to be able to cover parallelograms and not heeding the commands given. Square or rectangular units used to cover still have distance or exceed the circumference of the parallelogram. Five students use a strategy like this; every 2 people use type 1.1.a strategy, and 3 people use type strategy 1.1.b. After the interview, Joko who used strategy 1.1.a drew a unit square visually without using a concrete unit square, while Dika using strategy 1.1.b used a square of concrete units given. However, they are both confused to use a unit triangle, so the drawing is not structured.

*Visual covering*

The strategy in this type has characteristics where students use estimates to draw square or triangle units, so the unit size used varies and is estimated visually even though it is found that there are still areas not covered by parallelograms. Five students use a strategy like this. Strategy Type 1.2.c shows students making the least accurate estimation approach of the square unit size and given triangle. In the 1.2.d Strategy Type, students make a more accurate approach but only for unit squares, while this fails to estimate the unit triangle. Strategy Type 1.2.e students make an estimation approach to draw square or unit triangles, but Indah, who uses this strategy, claims that besides estimation, she also uses a concrete unit square to make it easier to estimate. Sinta and Lina who used the 1.2.c and 1.2.d strategies respectively claimed that they did not use concrete objects in unit drawing. This type of strategy also shows that when the estimations made by students are increasingly away from the actual size, students will fail in estimating the next unit, as Henki did in strategy 1.2.e.

*Concrete covering*

The characteristic of this strategy is that students use a congress square or triangle provided, and students can cover the entire parallelogram area. Eight students use a strategy like this, and this number is the most of the other strategies in task 1. Type strategy 1.3.f is the most emergent, in which students can correctly be structured using square and unit triangles, and they can accurately calculate the number of units needed. In the strategy type 1.3.g and 1.3.h, students use concrete covering when determining how many square units are needed, but tend to use visual covering when covering areas that must use a unit triangle. Indra, who used the 1.3.f strategy, was able to systematically explain the sequence of the closure of his area, albeit slowly. Ilmi, who uses the 1.3G strategy, can sort steps using a unit square and several triangles concretely, but because the unit size is not right, it confuses the remaining parallelogram regions and finally draws the unit triangle in an estimate. Dea who used strategy 1.3.h did almost the same thing as Ilmi, but the estimation that she did was wrong because it used a unit that was not given, namely rectangle.

*Measurement*

The characteristic of this type is that even though students use square or concrete unit triangles to initiate the closure of the parallelogram area, in the remaining areas that are generally covered in squares students realize there are long and wide multiplication patterns. Only 2 students can use this strategy. Erna and Elsa used the 1.4.i and 1.4.i strategies to realize that the square parallelogram area was 4x4 in size, in the process, they mentioned that they only had to use a ruler to draw it. To cover the remaining area, Erna uses a more ordered unit triangle estimation than Elsa.

***Intuitive Strategy on Task 2***

The characteristic of the strategy that appears in task 2 is based mostly on the estimation of the square and the unit triangle used to cover the parallelogram. Figure 6 shows the variation of strategies carried out by students.

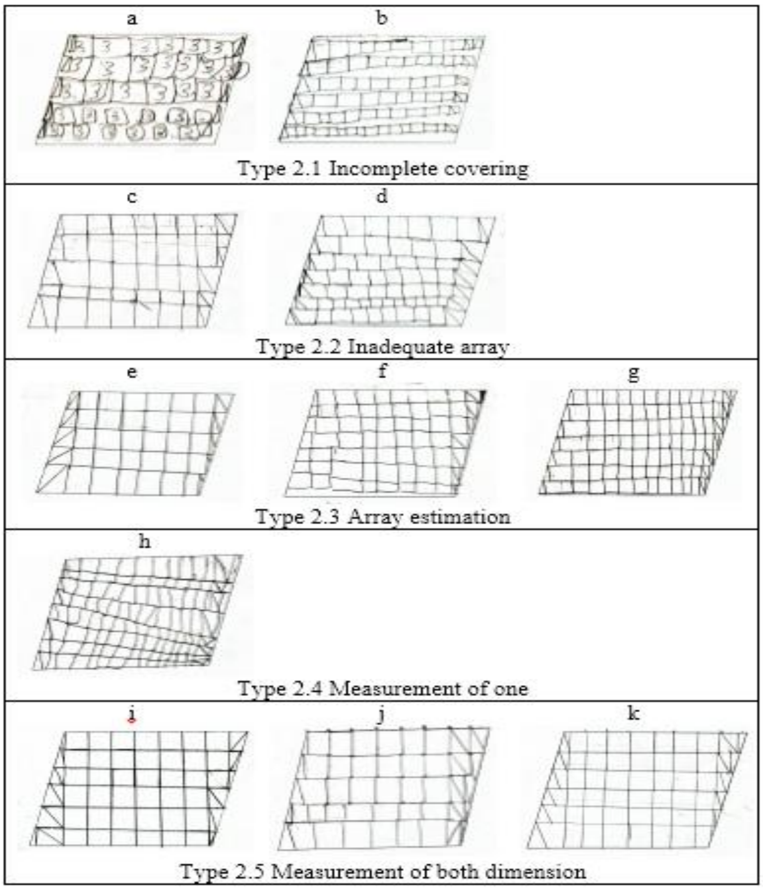


Figure 2. Result of intuitive Strategy on Task 2

*Incomplete covering*

The incomplete covering strategy in task 2 is not much different from the Incomplete covering strategy in task 1, where students failed to cover the parallelogram area. The square or triangle used exceeds or even less than the circumference of the parallelogram. But in terms of structure, the unit is more systematic than task 1. Four people who use this kind of strategy, Rina uses type 2.1.a strategy by marking a square unit with the number "3" because according to him, this makes it easier to distinguish unit. Three people use strategy type 2.1.b, including Henki. This strategy is similar to strategy 1.2.e that he previously did in Task 1. In Task 2, he could not estimate because he claimed there were no concrete objects that could help to cover the area.

*Inadequate array*

The characteristics possessed by this strategy is that students are more structured in covering the parallelogram area, but the unit used is very inaccurate or away from the actual unit size when making estimates. Six people can use this kind of strategy, Ikhsan who uses type 2.2.c strategy to do regional closure with the help of rectangle. This is inspired by the actual square unit size when making measurements in Task 1. Sinta is a student who uses the 2.2.d strategy type using the only estimation, but because the unit size is far from accurate, she still leaves an area that is not covered.

*Array estimation*

The main characteristic of this type of strategy is to use a simple estimation approach visually to cover the parallelogram. There are 6 students who answer using this strategy. The type 2.3.e strategy by Susi is the closest to accurate, Zori which uses the type 2.3.g strategy is almost the same as that done by Susi, but the unit accuracy is far from the actual size. Meanwhile, Lina who uses the type 2.3.f strategy still leaves areas that are not covered, and this strategy is similar to the type 1.2.d strategy in task 1.

*Measurement of one dimension*

This strategy has the characteristic of taking measures by paying attention to only one parallelogram aspect, which is only the base dimensions or height. Yopi is the only student to carry out this type of 2.4.h strategy. He intends to do a one-dimensional measurement parallel to the base of the parallelogram, but does not pay attention to the high dimensions and the unit estimates used are incorrect.

*Measurement of both dimensions*

This type of strategy is more complete and accurate compared to the measurement of one dimension strategy because students pay attention to two aspects of the parallelogram, namely the base and height. Most students, in this case, partition the parallelogram into 2 areas that have rectangular and triangular shapes, then estimate the triangle using unit triangles. Only 3 people can use this strategy. The type 2.5.i strategy used by Erna is the most accurate measurement, compared to Ilmi's 2.5.j type strategy, and Elsa's 2.5.k. Erna's strategy is similar to what she did when completing task 1 (see type 1.4.i strategy). Ilmi though uses the Measurement of both dimension strategy, but he tends to focus on the base dimensions so that measurements on high dimensions appear less systematic. He claimed to be inspired by assignment 1 because it involves concrete units, this strategy is similar to 1.3.g. While what Elsa did was almost the same as what she did herself in task 1 (see type 1.4.j strategy).

***Intuitive Strategy in Task 3***

Task 3 is only given to students who can use measurement strategies on task 2. In this task, students are only given the size. That is, the visual form of parallelograms, squares, and unit triangles is not shown. Most students who can use variations of this strategy are students who can make accurate estimates.

*Array estimation*

There are only 2 students who use array estimation strategies. This strategy used is the same as the strategy type 2.3 in task 2.

*Measurement of one dimension*

There is only 1 student who uses this strategy, the strategy used is similar to the 2.4 type strategy in task 2.

*Implied draw array*

Almost all students who use this strategy, partition parallelograms, such as type 2.5.j and 2.5.k strategies in task 2. So, student answers are only correct on the unit square needed to cover the rectangle, which is as much as 14 square units because the estimation is done wrong on the part of the triangle-shaped, so some students answer 10 unit triangles and some answer 12 unit triangles.

*Array is calculated*

There is only 1 student who can use this strategy. In addition to using the parallelogram area partition, as in the implied strategy array draw, he realized that to determine the number of squares or unit triangles needed, in addition to drawing, simply calculate using the multiplication concept or division that 8x8 = 64 then divide it by the square unit product namely 2 x 2 = 4 so that the unit square needed to cover the rectangle is 64: 4 = 14 square units. Meanwhile, the two triangular regions resulting from partitions are joined together, then the unit square that can cover is calculated as 16: 4 = 4. So, to cover the parallelogram, the number of square units needed is only 18.

The strategy variations of the results of this study are mostly similar to the results of the study (Lynne N & Michael C, 2000) which shows that in task 1 shows covering, visual covering, concrete covering, and measurement. Task 2 shows incomplete covering, inadequate arrays, array estimation, measurement of one dimension, and measurement of both dimensions. Meanwhile, on task 3 there is an array estimation, measurement of one dimension, implied draw array, and a calculated array. But there are differences in strategies raised by students in task 1, that there is a strategy where students combine visual covering and concrete covering. This type of strategy is biased if not grouped separately, then covering strategy concrete is a different strategy with pure visual covering and pure concrete covering.

The strategy variations that should be grouped separately are between inadequate arrays, array estimations, and measurement of both dimensions that found students who use array estimation appropriately because it involves measurement, whereas there are students who misuse array estimation. It is almost the same as the findings of other strategies in task 2. In the variation of task strategy 3, it also shows that there is another type of strategy when specifically seen between array estimation and measurement of one dimension. This means that there is another strategy that can be called measurement estimation strategy.

Visual-concrete covering and measurement estimation strategy findings in measuring the width of parallelograms using units that are not the only square, but also triangles indicate that broad measurement strategies are not only mentioned (Lynne N & Michael C, 2000). This has implications for the development of an intuitive understanding strategy that will be more detailed. Classification of strategy findings of this study at the level of development of intuitive understanding (Lynne N & Michael C, 2000) shows in Table 1.

Tabel 1. Classification of Strategy Intuitive Understanding

| **Developmental Levels** | **Task 1 Strategy** | **Task 2 Strategy** | **Task 3 Strategy** |
|---|---|---|---|
| Level 0: Incomplete covering | Incomplete covering | Incomplete covering | |
| Level 1:<br>Primitive covering | Visual covering | Inadequate array | Array draw |
| Level 2:<br>*Visual –concrete covering*<br>(Transition 1) | *Visual –concrete covering* | | |
| Level 3:<br>Array covering constructed from unit | Concrete covering | Array estimation | Array estimation |
| Level 4:<br>*Array covering constructed by measurement estimation*<br>(Transition 2) | | | *Measurement estimation* |
| Level 5:<br>Array covering constructed by measurement | Measurement | Measurement of one dimension | Measurement of one dimension |
| | | Measurement of both dimension | |
| Level 6: Array implied, solution by calculation. | | | Array draw implied |
| | | | Array calculated |

In Table 1. there are 6 levels of development of intuitive understanding that have more specific characteristics of a flat wake measurement strategy.

Level 0: *Incomplete covering*

Characteristics of this level of development, students are unable or fail to measure the area using the given unit. The strategy that might occur is also called Incomplete covering

Level 1: *Primitive covering*

Characteristics of this level of development, students take measurements of the area using a visual approach, but it is very inaccurate. Possible strategies include Visual covering, Inadequate arrays, and Array draw.

Level 2: *Visual –concrete covering*

Characteristics of this level of development, students take measurements of the area using a strategy combination between visual or concrete covering, but the estimation approach used is less accurate.

Level 3: *Array covering constructed from unit*

Characteristics of this level of development, students take measurements of the area using a more structured visual or concrete approach. Possible strategies that arise are Concrete covering and Array estimation

Level 4: *Array covering constructed by measurement estimation*

Characteristics of this level of development, students take measurements of the area using a structured and precise estimation approach.

Level 5: *Array covering constructed by measurement*

Characteristics of this level of development, students begin to realize that broad measures can depart from one dimension or two dimensions.

Level 6: *Array implied, solution by calculation*

Characteristics of this level of development, students can use calculations by using the concept of multiplication or division, with little drawing or without having to draw to determine the total unit area. A possible strategy is an implied draw array and a calculated array.

## CONCLUSION

In this study, it was found that there was an intuitive understanding strategy in measuring area size, which was found by (Lynne N & Michael C, 2000), namely visual-concrete covering strategy and measurement estimation. Given these findings, based on the characteristics of the development of intuitive understanding found a transition 1: visual-concrete covering and transition 2: covering constructed array by measurement estimation. So that it is seen from the structure of its characteristics, the level of development of the new intuitive understanding consists of level 0: incomplete covering, level 1: primitive covering, level 2: visual-concrete covering, level 3: array covering constructed from units, level 4: covering constructed by measurement estimation, level 5: constructed by measurement covering array, level 6: implied array, solution by calculation. Thus, a teacher can make a teaching and learning plan strategy more adequate to construct the area measurement for early elementary school students.

## REFERENCES

Ausubel, D. P. (1968). Facilitating meaningful verbal learning in the classroom. *Arithmetic Teacher*, *15*, 126–132.

Baranes, R., Perry, M., & Stigler, J. W. (1989). No Title. *Activation of Real-World Knowledge in the Solution of Word Problems. Cognition and Instruction*, *6*, 287–318.

Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Borrow, C. V. A. (1998). Students' Spatial Structuring of 2D Arrays of Squares. *Journal for Research in Mathematics Education*, *29*(5), 503–532. https://doi.org/10.2307/749731

Bennett, A. B., Burton, L. J., & Nelson, L. T. (2012). *Mathematics for Elementary Teachers: A Conceptual Approach*. New York: McGraw-Hill.

Berenson, S. B., & Carter, G. S. (1995). Changing assessment practices in science and mathematics. *Social Science and Mathematics*, *95*, 182–186.

Byers, V., & Herscovics, N. (1977). Understanding school mathematics. *Mathematics Teaching*, *81*, 24–27.

Creswell, J. W. (2012). *Educational Research: planning, conducting, and evaluating quantitative and qualitative research*. Boston, United States of America: Pearson Education.

Freudental, H. (1983). *Didactical Phenomenology of Mathematical Structures*. New York, London: Kluwer Academic Publishers.

Hiebert, J. (1981). Units of Measure : Results and from Implications National Assessment. *The Arithmetic Teacher*, *28*(6), 38–43.

Hiebert, J., & Carpenter, T. P. (1992). Handbook of research on mathematics teaching and learning. In *Learning and teaching with understanding. In D. A. Grouws (Ed.)* (pp. 65–97). New York: Macmillan.

Hirstein, J. J., Lamb, C. E., & Osborne, A. (1978). Student misconceptions about area measure. *Arithmetic Teacher*, *25*(5), 10–16.

Linchevski, L., & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability grouping in mathematics. Some issues in the psychology of mathematics instruction. *Journal for Research in Mathematics Education, 29*, 533–554.

Lynne, N, O., & Michael C, M. (2000). Young Children's Intuitive Understanding of Rectangular Area Measurement Young Children ' s Intuitive Understanding of Rectangular Area Measurement. *Journal for Research in Mathematics Education*, *31*(2), 144–167. https://doi.org/10.2307/749749

NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. Reston: VA: Author.

NCTM. (2010). Why is Teaching with Problem Solving Important to Student Learning? *National Council of Teachers of Mathematics*, *13*(12), 1–6. https://doi.org/10.1016/S2213-8587(14)70016-6

Outhred, L., & Mitchelmore, M. (2004). Students' Structuring of Rectangular Arrays. In *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 465–472).

Pirie, S. E. . (1988). Understanding: instrumental, relational, intuitive, constructed, formalized...? How can we know? *For the Learning of Mathematics*, *8*(3), 2–6. https://doi.org/10.2307/40248145

Skemp, R. R. (1987). *Psychology of Learning Mathematics*. Hillsdale, NJ: Erlbaum.