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WHY DID THE STUDENTS MAKE MISTAKES IN SOLVING DIRECT AND INVERSE PROPORTION PROBLEM?

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Abstract

The purpose of this study is to describe the student's difficulties in solving direct and inverse proportion problem. This research uses explorative qualitative research type. The subject of this research is the second semester student of Mathematics Education Study Program in East Java. Subjects were selected based on purposive sampling. The findings of this study are 86% of students are wrong in solving the problem of inverse proportion, 28% of students are wrong in solving direct proportion problem, and 91% of students are wrong in solving both problems in a single question. Then, the students who made mistakes in solving the problem were chosen purposively for interview. The finding in this research is the student(1) do not understand the use of variables, (2) do not understand the use of formulas, (3) do not understand the key phrases on the problem, (4) Difference in ratio, fractional, and division, (5) do not understand the problem, (6) do not understand simplification of division, and (7) do not interpret proportion relation correctly

Keywords: Mistake, Direct proportion, Inverse proportion

Fraction is one of the basic concepts of mathematics taught since elementary school (SD). At elementary level, students are taught various forms of fractions, fractional forms of operation and comparison (Hardi, Mikan, & Ngadiyono, 2009; Mustaqim & Astuty, 2008; Supardjo & Salamah, 2008). Students are first introduced fractions using discrete objects, such as pencils, marbles, chairs, etc.(Wu, 2013). If there are 4 marbles, one part of all 4 marbles can be written as ¼. Fractions have various shapes, namely: general fractions and mixed fractions. Ordinary fractions are rational numbers that usually written as $\frac{a}{b}$, with $b \neq 0$, while the mixed fraction is in the form of $c\frac{a}{b}$. Understanding fraction can be done with concrete objects, and fractions are the basis for learning ratios and comparisons (Lobato, Ellis, Charles, & Zbiek, 2010; Lobato, Orrill, Druken, & Jacobson, 2011).

In its use, fractional forms can be used to represent ratios and divisions (Van Galen et al., 2008; van Galen & van Eerde, 2013; Wu, 2011). For example, there are fractions $\frac{2}{3}$, can be used to represent the ratio of 2: 3 *(part-whole)* and also $2 \div 3$ (2 divided by 3). To distinguish the fractional form between ratios and divisions requires a deep understanding. If students have a deep understanding, then the concept of the ratio and the division in the form of fractions will not be interrupted. In fact, students experience interference and are weak in understanding ratios and comparisons (Lamon, 2006). As a result, he/she cannot solve the problem given. Weak understanding of fractions can cause problems when understanding proportional issues (Irfan, Sudirman, & Rahardi, 2018; Jordan et al., 2013; Sadler & Tai, 2007).

The importance of proportional reasoning has been put forward by many researchers, including (Arican, 2016; Boyer & Levine, 2012; Irfan et al., 2018; Lamon, 2006; Nagar, G. G., Weiland, T., Brown, R. E., Orrill, C. H., & Burke, 2016; Sumarto, Van Galen, Zulkardi, & Darmawijoyo, 2014; Walle, Karp, & Bay-Williams, 2010). Proportional reasoning is important in understanding many situations in science and in everyday life (Arican, 2016; Son, 2013). Proportional reasoning describes various types of reasoning that focus on the relationship between two ratios and requires complex ideas. According to Boyer & Levine (2012) proportional reasoning requires some understanding of the relationship of scale and appears i n everyday problems. Proportional reasoning includes the equivalence of fractions, divisions, place values, percentage calculations, and measurement conversions (Lobato et al., 2010). Fractions and ratios are two important concepts in proportional reasoning. Understanding of mathematics teacher candidates about ratios and fractions should be strong and coherent. They must understand that the ratio is the ratio of two quantities(Lamon, 2006) whereas a fraction of part-whole relationships *(part-whole relationship)*(Brown, Nagar, Orrill, Weiland, & Burke, 2016; Nagar, G. G., Weiland, T., Brown, R. E., Orrill, C. H., & Burke, 2016). Frith & Prince (2016) state that proportional reasoning contains two problems, namely the problem of comparison and problems finding an unknown value*(missing value problem).*

Proportional reasoning has been widely studied. According to (Lemonidis, 2008; Sumarto et al., 2014; Walle et al., 2010) many teachers only teach crosscutting procedures to solve comparative problems. Son (2013) mentions that students do not understand the meaning of cross multiplication. (Fernández, Llinares, Van Dooren, De Bock, & Verschaffel, 2012; Van Dooren, De Bock, Evers, & Verschaffel, 2006) stated that students often use the comparison method for integer ratio problems, and more use the addition method for non-integer ratios. Students use the comparison method on non-comparison problems, and use non-comparison methods on comparison problems(Van Dooren, De Bock, Evers, & Verschaffel, 2009; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2004). The results of Van Dooren et al. (2009) suggest that non-comparison problems with non-integer ratios cause students to use the concept of comparison in solving the problem. (Lemonidis, 2008)in his research found that the students experienced pseudo-proportion when creating an example of a problem related to real life.

Based on the description of the problem, it appears that proportional reasoning is one of the mathematics that is difficult to understand by individuals. From the various studies that have been done, researchers try to describe the difficulties that occur in prospective students when the teacher to solve the direct and inverse proportion problem.

METHOD

This research uses qualitative-explorative research type. The researchers tried to explore the thinking process of students in solving proportion problems, and then traced the difficulties encountered when solving the problem. Research subjects were chosen using purposive sampling technique chosen from the second semester students of Mathematics Education Study Program in East Java in academic year 2016/2017 consisting of 43 students. The main instrument in this study is the researcher himself and its supporting instruments are related to fractions and proportion worksheets, interviews, audio-visual recordings.

Subjects were given worksheets related to fractions and proportions. There are two problem-solving questions, which can be seen in Table 1.

Table 1. Problem-solving question aboutproportion

Types of Problems	Question
Direct Proportion	Sofyan wants to travel from Kedungkandang to Batu as far as 80 km, with an average speed of approximately 60 km / hour. Because there is an urgent need, he must arrive at Batu 30 minutes faster. What is the average speed that is needed by Sofyan?
Inverse Proportion	Sofyan vehicles can accommodate 20 litres of Pertamax and can travel up to 450 km. How many litres does Sofyan need to travel as far as 112.5 km?

After students do the questions, then the work is checked. Then the researcher chose a student whose answer is wrong in both final answer and the process. In the end, the researcher chose two students to be the subject of research. Researchers chose two subjects with consideration of student worksheets as well as students' communication skills. After obtaining the subject of research, researchers interviewed and recorded its both audio and video.

RESULT AND DISCUSSION

Of 43 students of the second semester of the Mathematics Education Study Program of academic year 2016/2017, only 14% of them answered number 1 correctly and 72% answered question number 2 correctly. The complete details can be seen in Table 2.

Table 2. Recapitulation of Student's Answers

Question	Participant Answers				amount
	B	Percentage	S	Percentage	
1 (inverse proportion)	6	14%	37	86%	43
2 (direct proportion)	31	72%	12.	28%	43

Based on Table 2, it shows that students have difficulty to solve the problem of inverse proportion. However, it turns out that on the question of direct proportion, there are still a lot of students who gave wrong answers. This becomes a serious problem, because they are students of education in which they will teach their future students.

The researchers conducted interviews to find out in depth about the work of subject 1 (S1) and subject 2 (S2).

- Subject 1 (S1)

The work of Subject S1 onquestion number 1 is wrong, while in question number 2 and 3, he answered correctly. In here, researchers describe the work of the subject S1 only on the question number 1. The work of Subject S1is shown in Diagrams 1 and 2. Based on the answer of Subject S1 and interview that were conducted by the researchers, it showed that subject S1 is not able to understand the problem correctly. He defined phrase *"he shall arrive at Batu 30 minutes faster"* with the meaning of time it takes is 1 hour - 30 minutes = 30 minutes. "1 hour" is taken from the average speed that Sofyan needs, i.e. 60 km / h. He did not consider Sofyan's mileage.

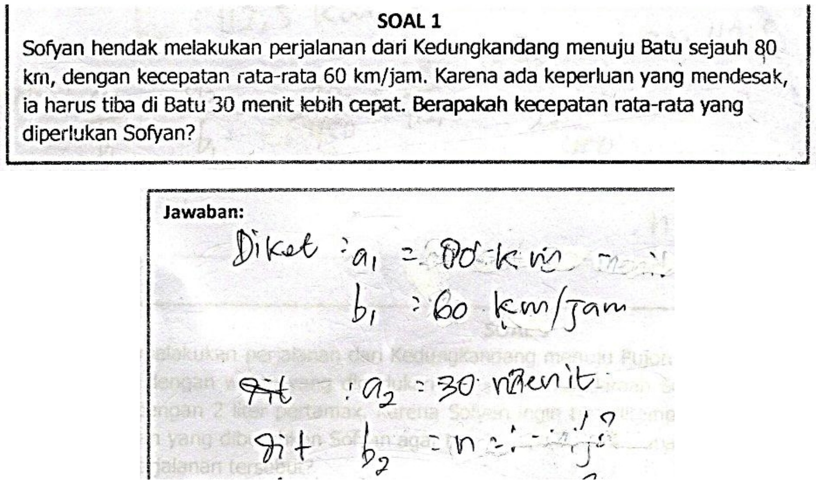


Figure 1.Subject S1 work on question 1, particularly in the part of understanding the problem

Researchers then conducted an interview to the subject as mentioned below

Researchers	:	What is the meaning of*a1* and *b1*?
Subject	:	*a1*isSofyan's mileage and *b1 isspeed.*
Researcher	:	What if*a2* and *b2?*
Subject	:	*a2*is the time that is required bySofyan and *b2*is the required speed.
Researcher	:	Thus, is the letter*a*, both*a1* and *a2* different?
Subject	:	It is different sir.
Researcher	:	You wrote *a2* = 30 minutes. What does it mean?

Subject : Time needed by Sofyan. She arrived 30 minutes faster.

Based on these conversations, it is indicated that the subject S1is not consistent for writing *a* variable to represent the distance or required time. In addition, the subject S1 is also not able to interpret the phrase "he must arrive 30 minutes faster", so he wrote down the time it takes only 30 minutes, should be 50 minutes. Subject S1 does not look for travel time gained from the distance of 80 km and speed 60 km / hour. Therefore, Subject S1 cannot understand the problem correctly. It is also difficult to write into a mathematical model, especially the representation of a variable.

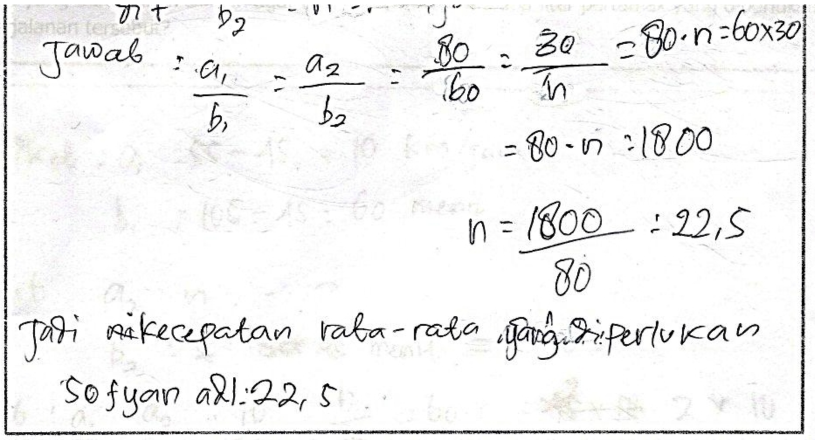


Figure 2. Subject S1 work on problem 1, the problem solving section

Researcher : You wrote $\frac{a_1}{b_1} = \frac{a_2}{b_2}$. What does 'division' symbol mean?

Subject : As far as I know, if it is about the symbol, I do not really understand. However, to search this, use this formula.

Researcher : What kind of formula is that?

Subject : Statistics

Researcher : Statistics? In terms of? Statistics formulas are many, there is average, mode, median...

Subject : (silent)

Researcher : This, 80 per 60 means 80 divided by 60 or 80 per 60 or what does it mean?

Subject : 80 per 60.

Researcher : Meaning?

Subject : (Silent)......... .. That means the other words?

Researcher : Yes, it can be like that.

Subject : 80 per 60 is a fraction

Researcher : Oh, so 80 per 60 is a fraction?

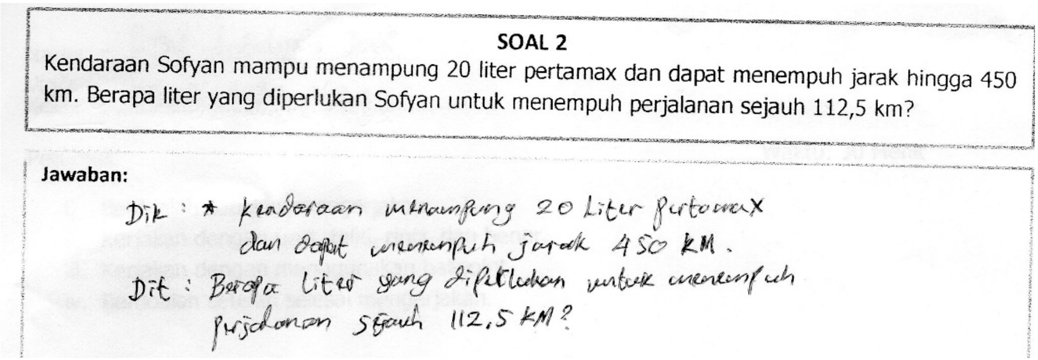
Subject : Yes sir.

Furthermore, in the process of solving problems and interpretation of solutions obtained, the subject S1 does not interpret the form correctly. He considered form Are two pieces of the same value. This finding is consistent with the results of research conducted by(Jordan et al., 2013; Sadler & Tai, 2007; Son, 2013). In addition, the subject S1 is also unable to explain why he is using to solve the problem. Means, subject S1 in solving the problem only procedural. This is in accordance with the results of research that has been done by (Lemonidis, 2008; Son, 2013; Sumarto et al., 2014).

- Subject 2 (S2)

The subject of S2 work on questions 1 and 3 is working correctly, but on item 2, he is wrong. Thus, the researcher describes the work of the subject S2 on problem number 2 only. Based on the work of the S2 Subject, the researcher conducted an in-depth interview. S2 Subject Work can be seen in Figures 3 and 4.

What is interesting about the subject's work is that he does not use the variables when writing down what is known and asked from the question. He writes the premises that are important, such as the *"vehicle holds 20 litres"* and *"can cover a distance of 450km"*. Later, he wrote the sentence asked *"how many litres are required to travel as far as 112,5km"*.


SOAL 2

Kendaraan Sofyan mampu menampung 20 liter pertamax dan dapat menempuh jarak hingga 450 km. Berapa liter yang diperlukan Sofyan untuk menempuh perjalanan sejauh 112,5 km?

Jawaban:

Dik : * kendaraan menampung 20 Liter pertamax dan dapat menempuh jarak 450 KM.

Dit : Berapa liter yang diperlukan untuk menempuh perjalanan sejauh 112,5 KM?

Figure 3. The work of Subject S2 on problem 2, in terms of understanding the problem

Researchers	Why are you writing sentences, not using variables?
Subject	It's easier this way sir.
Researcher	Can it be with a variable?
Subject	It could be sir.

Based on the answer sheet and the interview quote, the subject S2 shows that he is easier to write with sentences than using variables. During the process of solving the problem, he immediately wrote down the similarities and wrote the variable x for the unknown value.

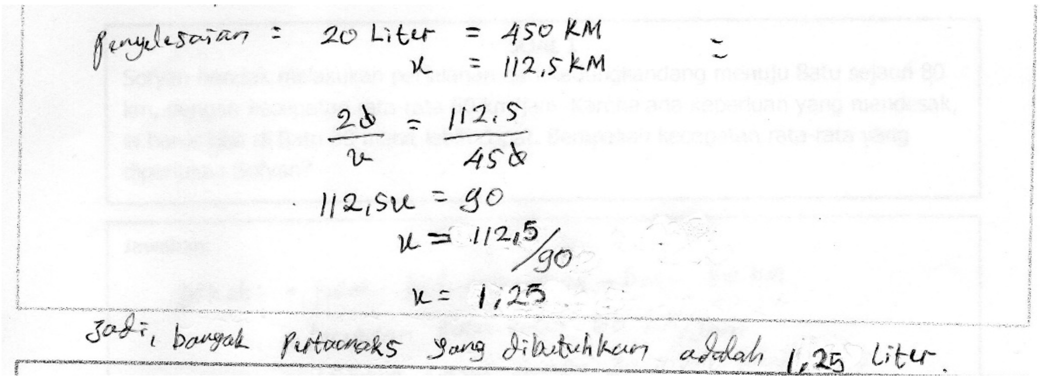


Figure 4. The work of the subject S2 on problem 2, the problem solving section

Researcher	What do you mean by 20 litres = 450 km?
Subject	That is, 20 litres can be for a distance of 450 km.
Researcher	If x = 112, 5 km?
Subject	How many liters for a distance of 112.5 km.
Researcher	Why did you write $\frac{20}{x}=\frac{112,5}{450}$?
Subject	To find the value of x pack. How many litersareneeded.
Researcher	Should the sequence be like that? Must be 20 at the upper left, lower left x, 112, 5 in the upper right, and 450 in the bottom right?
Subject	Yes sir. 20 liters for 450 km, and x for 112.5 km.
Researcher	Oh, yes. Why did you scratch the number 0 in the 20 and 450?
Subject	It can be simplified sir.
Researcher	Why?
Subject	Because 20 is above, and 450 is below.

Based on the work result of S2 subject in Figure 4 and interview, it was found that the subject of S2 was unable to compile the comparison correctly. He wrote down a comparison $\frac{20}{x}=\frac{112,5}{450}$ which is a turnaround ratio, he should write $\frac{x}{20}=\frac{112,5}{450}$. Because he was wrong in writing comparisons, consequently the answer was wrong. In addition, he does not understand the simplification of the division. In a sense, the above numbers can always be simplified with the numbers below. This is seen when the subject S2 simplifies 20 by 450. He scores the number 0 on each number. Thus, he obtained a 1.25liters answer. If the comparison done correctly, it will produce the value x = 80 liters.

The subject of S2 uses the concept of reversal of value to resolve the problem of comparative worth, this is called interference. This is in line with Anderson, (2003); Anderson & Neely(1996). Interference occurs when there is difficulty in remembering an object because of the similarity of objects stored in memory(Anderson, 2003; Anderson & Neely, 1996; Slavin, 2006; Sternberg & Sternberg, 2011).

Based on the results of the work of both subjects, the results obtained are presented in Table 3.

Table 3. Causes of mistakes S1 and S2 Subjects

No	S1	S2
1	Do not understand the use of variables	Did not understand the problem
2	Do not understand the use of formulas	Did not understand the simplification of the division
3	Did not understand the key phrase on the problem	Does not mean the comparison relationship correctly.
4	Do not understand the difference in ratios, fractions, and divisions.	

CONCLUSION

Based on the results of the study, it can be concluded that the causes of errors in solving the problem of comparative worth and turning values are: (1) do not understand the use of variables, (2) do not understand the use of formulas, (3) do not understand the key phrases on the problem, (4) difference in ratio, fractional, and division, (5) do not understand the problem, (6) do not understand simplification of division, and (7) do not interpret proportion relation correctly. Based on these findings, this research can be developed about the process of thinking interference in solving the problem of direct and inverse proportion.

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