ISSN 2615-2495

International Journal of Insights for Mathematics Teaching
Volume 01, No. 1, March 2018, pp. 35-43

THE PROCESS OF DISCOVERING STUDENT'S CONJECTURE IN ALGEBRA PROBLEM SOLVING

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Abstract

This exploratory descriptive research aims to describe the process of discovering student's conjecture in mathematics problem solving. There were 2 students in grade VII of Junior High School who participated as the research subject. The instruments used in this research were problem solving test and interview. This research consisted of three stages which were: 1) data collection; data taken process where the researcher asked every student to solve the problem given; 2) analysis on students' work and interview; in this step the researcher analyzed the results of the students' work and carried out interview with the students for further examination of conjecture discovering process when solving the problem; and 3) examining and concluding students' work result and interview result. The result of this study shows that the stages in discovering conjecture were done sequentially although not all steps were done

Keywords: Conjecture; Problem solving; Mathematics

Conjecture and problem solving are related each other since conjecture is involved in mathematics problems solving. This statement is in line with the opinion of some experts(Cañadas, Deulofeu, Figueiras, Reid, & Yevdokimov, 2007; NCTM, 2000; Yeo & Yeap, 2010). (NCTM, 2000) explains that working in mathematics involves a discovery, whereas the key of discovering something is through conjecture. Additionally, Canadas& Castro (2007)suggests that problem solving and conjecture discovery are mathematical activities that are intertwined in many ways. Yeo & Yeap (2010) further state that discovery process occurs in problem solving, furthermore, formulating and conjecture testing are the steps of investigation process.

The definition of conjecture itself has been explained previously by some experts (Canadas & Castro, 2007; Cañadas et al., 2007; Mason, Burton, & Stacey, 1982; Sutarto, Nusantara, Subanji, & Sisworo, 2016). (Canadas & Castro, 2007) explain that conjecture is a statement based on empirical fact that has not been validated. Conjecture is a statement about all possible cases based on empirical fact with an element of doubt (Canadas & Castro, 2007). Moreover,(Mason et al., 1982) argue that conjecture is a logic statement, but the validity cannot be guaranteed yet. According to (Sutarto et al., 2016)conjecture is a statement based on empirical fact with an element of doubt. Based on the opinions stated above, it can be concluded that conjecture is a statement (oral or written statement) from logical thinking process, yet the validity cannot be guaranteed yet.

In addition to their statements,(Cañadas et al., 2007) states that there are 5 types of conjecture, they are 1) type 1: empirical induction from a finite number discrete case which has consistent pattern to be observed, 2) type 2: empirical induction from dynamic case; conjecture can be made of a general rule that describes the nature of a set of dynamically related events, 3) type 3: analogy; conjecture can

be made by analogy to something already known fact, 4) type 4: abduction, conjecture can be made of a general rule that would explain an otherwise inexplicable case, sample, or events, and 5) type 5: perceptually based conjecturing; a conjecture can be made from a visual representation of a problem or a perceptual translation of its statement.

Cañadas et al, (2007)further explains some stages of each conjecture type. Type 1, empirical induction from a finite number discrete case has 7 stages, they are: 1) observing cases, 2) organizing cases, 3) searching for and predicting pattern, 4) formulating a conjecture, 5) validating the conjecture, 6) generalizing the conjecture, and 7) justifying the generalization. In addition, type 2, empirical induction from dynamic case has 6 stages; they are 1) manipulating a situation dynamically through continuity of cases, 2) observing an invariant property in the situation, 3) formulating a conjecture that the property holds in other cases, 4) validating the conjecture, 5) generalizing conjecture, and 6) justifying the generalization. Meanwhile, type 3, conjecturing by analogy could proceed through these 6 stages, they are 1) observing two cases, 2) searching for similarities between the cases, 3) formulating a conjecture based on the similarity, 4) validating the conjecture, 5) generalizing the conjecture, and 6) justifying the generalization. Furthermore, type 4 which is conjecturing by abduction involves 6 stages, those are: 1) observing one case, 2) observing a surprising or significant feature of that case, 3) formulating a conjecture that the feature applies to other cases, 4) validating the conjecture, 5) generalizing the conjecture, and 6) justifying the generalization. And the last, there are 7 stages in type 5, perception based conjecturing, as follows: 1) translating the problems into a perceptual representation, 2) constructing a personal mental representation of the mathematical elements involved, 3) perceptually observing special features of the representation, 4) formulating a conjecture based on the special features of representation, 5) justifying or formalizing the translation, 6) generalizing the conjecture, and 7) justifying the generalization. From those steps in discovering conjecture, the researcher summarized some stages to discover conjecture used in this research, those are: 1) observing the problem, 2) discovering and predicting the pattern, 3) formulating conjecture, 4) validating conjecture, 5) generalizing conjecture, and 6) justifying generalization.

There are many researchers who significantly contribute toward this conjecture topic(Aaron & Herbst, 2015; Cañadas et al., 2007; Garcia, Benitez, & Ruiz, 2010; Lee & Sriraman, 2011; Lesseig, 2016; Lin, 2016; Lin & Tsai, 2013; Reid & Jniversitv, n.d.; Sutarto et al., 2016). Lin & Tsai(2013)designed a project to discover conjecture and provided evidence involving elementary school students. Cañadas, et al (2007) suggests considering some factors in problems that allow or inhibit conjecture, types of conjecture, and the implications for teaching. Lee & Sriraman, (2011)explain that in open classical analogy problem solving, students explore similarities of perceptual, transitional, and relational. (Garcia et al., 2010)documented and analyzed university students' method in using some representations to make and verify the conjecture. While Lesseig, (2016)developed "Studio of Mathematics" to improve teachers' ability to discover Mathematics

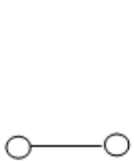
conjecture, generalize, verify and design learning activity that give students chance to solve problems of generalizing pattern based on APOS. In addition, Lin, (2016)helped teachers in creating conjecture project so the students can be actively involved in mathematics discussion activity. Moreover, Aaron & Herbst (2015)examined teachers 'perception toward students' attitude in geometer material in creating conjecture. The last, Reid & Jniversitv (2002)describes one pattern of rationalizing that is observed in mathematics learning process in a particular school in grade 5.

Those researches mentioned above show that the process in discovering conjecture is essential to be examined further because the process in discovering conjecture is the most important part in the process of problem solving. However, the researchers conducted previously were only limited on examining the process of students' conjecture discovery in problem solving of generalizing linear pattern of graphic. As a matter of fact, the process in discovering students' conjecture in mathematical problems is not only about linear pattern generalization but also squared pattern generalization. Hence, this research is important to be taken into account in order to describe the process of discovering students' conjecture in mathematics problem solving of squared pattern generalization.

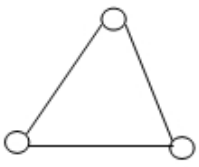
METHOD

This research used exploratory descriptive method. The research was conducted at 1 April 2017 and there were 2 research subjects. The research subjects were 2 students of grade VII in Junior High School. The instruments used in this research were problem solving test and interview. The question items used in this research were compiled based on cases or events to explore students in discovering conjecture. The level of difficulty given was for intermediate level since the subjects chosen have intermediate ability in mathematics. These examples below are the questions used in the research:

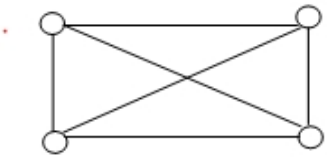
> Every Ied Mubarak, Islamic Boarding School Gontor Putri will hold Halal Bi Halal event. The students of Islamic Boarding School are 185 where every student will do one shake hands with the other students. The process of shaking hands is illustrated in Picture-1, Picture-2, and Picture 3 below.



Picture-1



Picture-2



Picture-3

Note

⬭ = Student, ——— = Shaking hand

Guideline:

1. Picture-1 shows that there are 2 students shaking hands at once.
2. Picture-2 shows that there are 3 students shaking hands three times.
3. Picture-3 shows that there are 4 students shaking hands six times.

Questions

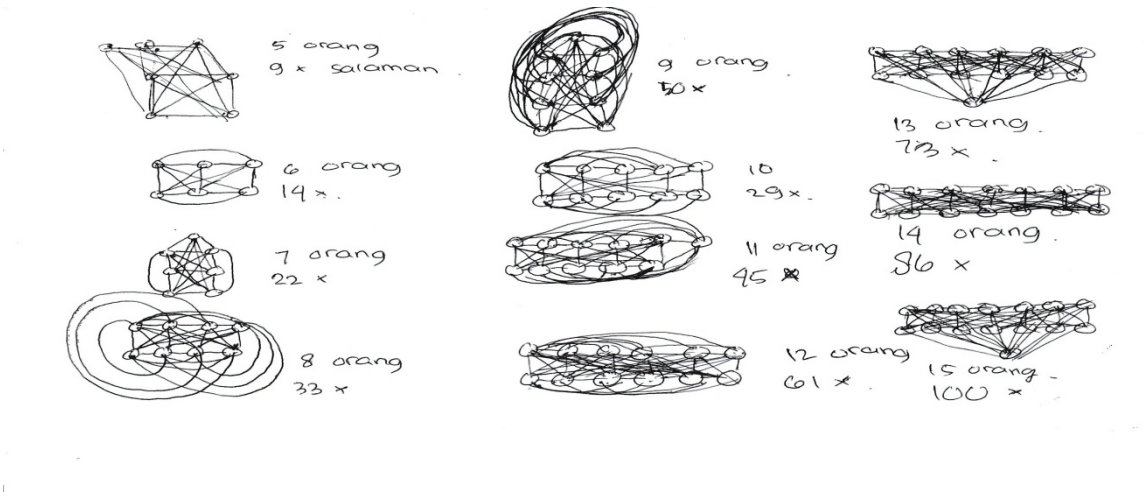
1. Decide how much handshaking is done by 5 students, 6 students, 7 students, until 185 students.
2. Find the general formula to decide how much hand shakingis done by the n-student. Explain how you find the formula.

This research consisted of three stages, they were: 1) data collection stage; in this stage, the researcher asked every student to finish problem given. While the students were solving the problem, the researcher did not interfere or involve in any interaction with student; 2) analysis on students' work and interview stage; in this stage, the researcher analyzed the result of students' work and carried out interview with the students to examine further about the process of discovering students' conjecture in solving problem; and 3) examining and concluding the result of students' work and of the interview.

RESULT AND DISCUSSION

Based on the result of the students' work analysis and interview, it is obtained a picture of discovering students' conjecture process in problem solving that will be explained more in each subject, as it follows:

Subject 1 (S1)



Picture 1. The Working Result of S1

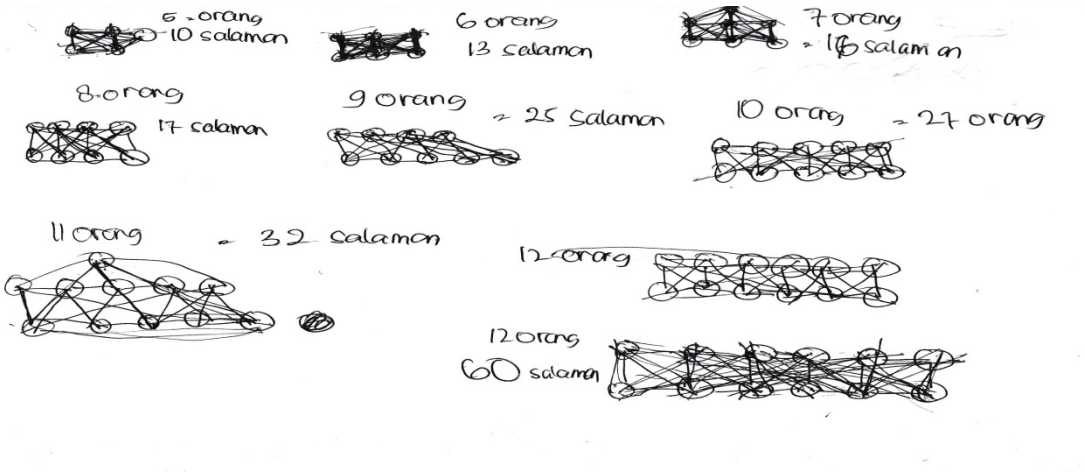
Based on Picture 1, it is shown that S1 found the numbers of handshakes done by 5 people by using the 3rd picture and adding 1 dot above it. Then S1 connected the four dots to the 3rd picture. The right answer of the handshakes number is actually 10 handshakes, yet S1 answered 9 handshakes. It happened because S1 made mistake in counting the numbers of handshakes. In finding the numbers of handshakes done by 6 people, S1 also used the 3rd picture by adding two dots beside it then connected them, so the result gained was 14handshakes. This same method was used by S1 in finding the numbers of hand shakes done by 15 people. Therefore, S1 made mistake in counting the numbers of hand shakes since S1 connected the dots one by one.

Based on the interview with the subject S1, it was found out that S1 observed the case given then explored and predicted pattern by looking at the previous picture to find the second picture, while S2 used the 1st picture. For instance, when there were 2 people shaking hands, it means there was 1 handshake. Another example is when there were 3 people shaking hands; it means there were 3 handshakesoccurred. In deciding the answer of 5 people shaking hands, S1 looked the previous picture, which was there were 6 handshakes done by 4 people. So S1 added 1 person and connected it (1 person was connected to the previous four people) in finding the answer. It shows when there were 5 people shaking hands, there were 9 handshakes happened. Even though there was a difference between interview and the result of S1's work, S1 stated that for 6 until 15 people counted, S1 added one person and connected it to the previous person to predict the pattern. Here is the transcription of interview between the researcher and subject S1:

R : How do you find the numbers of handshakes done by 5 people?
S1 : Well, at first, we have 4 people. When there are 4 people, it means there are err…6handshakes. So, when we have 5 people, we just need to add one person here and we connect them so we get 9 handshakes.
R : So, you will do the same when we have 6 people?
S1 : Well, yeah… We just need to see the previous picture.
R : Are you sure about this?
S1 : Yes, I guess so. (smiling)

From the questions delivered, it can be concluded that, S1 had tried to formulate the pattern by answering orally and tried to validate the statement. However, S1 did not try to prove the theory to the next phase (the n-person). It happens since S1 found difficulty in imagining and formulating conjecture until the n-person.

Subject 2 (S2)



Picture 2. The Working Result of S2

Based on the Picture 2, S2 found the numbers of handshakes done by 5 people by using the 3rd picture. S2 added one dot beside the 3rd picture then connected the four dots to the 3rd picture. The numbers found was exactly 10 handshakes. In finding the numbers of handshakes done by 6 people, S2 also used the same method by using the 3rd picture and added two dots beside it. After that, the dots were connected so the answer gained was 13 handshakes. Unfortunately, the answer was incorrect. It happened due to the error in counting the numbers of handshakes. This same method was used by S2 until finding the numbers of handshakes done by 12 people. S2 made a mistake in counting the numbers of handshakes because S2 connected the dots one by one.

Based on the interview with S2, it was discovered that S2 observed the case give then explored and predicted the pattern by drawing the numbers of people (the circle) then connected them one by one. For example, when there were 5 people, S2 drew 5 circles then connected them one by one so it gained 10 handshakes. Here is the sample of interview between the researcher and subject S2:

R : How do you find the numbers of handshakes done by 5 people?
S2 : So, I made 5 circles, and then one, two, three, four, five, six, seven, eight, nine, ten, and for the 6 people, I made one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, and for 7 people, one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen.
R : How do you find the numbers of handshakes of 8 people?
S2 : Well, it is same, just like the previous method.

In finding the numbers of handshakes of 6 until 12 people, S2 formulated the pattern by connecting one by one, so there was an error in finding the numbers of handshakes. S2 did not continue to the next phase (the n-person) because it was hard for S2 to predict and formulate the pattern of conjecture until the n-person.

Based on the working result and interview of the students, it is obtained the steps in discovering conjecture in mathematics problems solvingthat is explained in Table 1, as it follows:

Table 1. Steps in Discovering Conjecture in Problem Solving

Steps in Discovering Conjecture	Subject	
	S1	S2
Observing problem	√	√
Exploring and predicting pattern	√	√
Formulating conjecture	√	√
Validating conjecture	√	-
Generalizing conjecture	-	-
Justifying generalization	-	-

Based on the Table 1, it is shown that the first activity performed by subject S1 and S2 in discovering conjecture was observing problem. Observing problem is the first activity to determine the success of the students in solving problem. This opinion is congruent with Sutarto, Toto Nusantara, Subanji, (2016)stating that observing problem is the first activity done toward particular problems from the proposed problems. The next activity conducted by Subject S1 and S2 was exploring and predicting the pattern. According to Reid & Jniversitv (2002)exploring and predicting pattern is similar as "observing pattern" and even it goes beyond simply observing a pattern. When someone observes a repeated and regular situation, one naturally imagines that the pattern might apply to the next unknown case. Subject S1 and S2 tried to formulate conjecture from the case given. Formulating conjecture means making statement about all possible cases, based on empirical facts, but with an element of doubt (Cañadas et al., 2007). Only subject S1 who carried out validating conjecture and ensuring that conjecture obtained is correct. In addition, Sutarto, Toto Nusantara, Subanji(2016) also states that validating conjecture is an activity conducted to validate conjecture resulted from some particular cases which are not general.

Subject S1 and S2 did not continue to the next step, which is generalizing conjecture because they could not formulate conjecture until the n-person. Moreover, they also found some difficulties. It is congruent with the research of Sutarto saying that the number of students who find difficulties in conducting the stages of discovering conjecture process is 82.86%. This difficulty occurred because teachers rarely give questions related to this, so the students are not experienced and not used to do those problems. Besides, the students also get low score because of inappropriatelearning factorfor international standard so material tested in various evaluations is insufficient. Thus, it leads to the students' failure when they take the test (Novita, 2012). Based on international evaluation, such as *Program for International Student Assessment* (PISA) and *The Third International Mathematics and Science Study* (TIMSS), they show that the level of ability of Indonesian students in problem solving

is poor, especially in solving non-regular or high level problem (Stacey, 2011). These evaluations show that Indonesian students' ability is still poor so it is still hard to solve problem well.

The process in student's discovering conjecture points out that students' thinking process in solving problems is based on stages of discovering conjecture process in empirical induction from finite number of discrete case. According to the research of Lin & Tsai(2013) saying that in empirical induction from finite number, conjecture can be made based on observation of finite number of discrete case, in which the pattern can be observed consistently. This type of conjecture is often found in problems involving numbers, yet in some cases, not all conjectures can be proved in mathematics induction though a general pattern has been discovered.

CONCLUSION

Based on the research findings and discussions, there are 4 stages of process in discovering conjecture, there are: 1) observing research problem, 2) discovering and predicting pattern, 3) formulating conjecture, and 4) validating conjecture. These stages were done by the students in sequence, though not all steps were done. In the step of generalizing conjecture, the students encountered a problem since they are not used to do non-routine questions and not experienced in solving that problem, especially conjecture. However, this research is lack of expert validation for the solving problems given. Therefore, it is expected for the future researchers to increase the research subjects, so there will be more students who do all steps in discovering conjecture. Furthermore, the future researchers should consider the ability of the subjects so they will do all steps to discover conjecture.

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