ISSN 2087-8885

International Journal of Insight for Mathematics Teaching
Volume 01, No. 2, October 2018, pp. 130-140

INFORMAL STATISTICAL REASONING OF STUDENTS TAKEN FORMAL STATISTICS LEARNING RELATED TO DISTRIBUTION

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Abstract

This research aims at describing students' statistical reasoning in graphics statistics representation related to distribution. The subjects of this research were students of semester IV of Program of Study Mathematics Education of Muhammadiyah University Ponorogo who have taken a basic statistics course. These subjects were chosen because they have taken a course related to descriptive statistics which discusses graphics representation and data distribution. The data collection technique used essay test related to graphics representation. In addition, an interview was also conducted to confirm students' answer. This research finding shows that statistical reasoning of semester IV students of Mathematics Education whose statistics ability is poor belong to pre-structural level and whose statistics ability is high belong to multi-structural and relational level. The high skilled students could conclude the data with statistical reason even though they used informal terms. Students with multi-structural and relational level regard that variety is a standard showing the numbers of different data among others, not on the different value from the average. Students with relational reasoning level were able to generate graphics concluding by connecting the central tendency and distribution scale.

Keywords: statistical reasoning, informal, distribution

Statistics become the main concern in Mathematics education discipline for the last decades. It occurs since statistics are closely related to daily life problems. Moore (1997) believes that statistics is a means to solve problems in daily life, either in working circumstance or scientific business. It is in line with Zieffler, A., Garfield, J., Delmas, R., & Reading, C. (2008) who state that making the conclusion of data obtained is a part of our life, furthermore critical observation of the statistics conclusion is an important part for most students taking the introduction of statistics.

In the learning process, there is no adequate concern for statistical reasoning. The Standard Curriculum in developed countries is less focusing on data analysis in the first years, yet they concern more on numbers, measurement and geometry as it is explained by Ginsburg & Leinwand (2009). The other characteristics can be seen from the statistics book content which tends to guide students to calculate rather than reason. Mokros & Russell (1995), Fieldman, A., Konold, C., Coulter, B., & Conroy, B. (2000), Konold & Higgins (2002), Lemsan & House (2012), Jacob (2013) assume that statistics literature existing now prioritize more on procedural success rather than conceptual understanding. Sedlmeier (2002) says that statistical reasoning is rarely taught to the students. Even it is rarely successful when it is taught. Furthermore, Ben-zvi & Garfield (2004) explain that basically students have good statistics learning, obtain a good score for take-home assignment, test, and other assignments, yet they are still poor in statistical reasoning. The effects of learning focusing on procedural knowledge can be seen in PISA and OECD 2014 which show that the students find difficulties in assignments related to the using, interpretation, data reasoning related to the real world.

Chance & Garfield (2001) state that students with good statistical reasoning are able to make the connection and expressing the relation among statistic concepts. delMas (2002) argues that statistical reasoning allows us to explain a statistic process such as why and how a conclusion is drawn. Lovett (2001) interprets statistical reasoning as statistic instruments and concepts to review, predict, and conclude the data. Those above-mentioned opinions are congruent with Shaugnessy, J. ., Chance, B., & Kranendonk, H (2009) who define that students' statistical reasoning as an activity of constructing questions, designing and employing plan to collect data, analyze and review data, and interpret the result of data analysis.

This statistical reasoning model has been developed by some experts; they are Shaughnessy, J.M., M. Ciancetta, K. Best and J Noll (2005); Reading dan Reid (2006). Biggs & Collis (1982) categorize thinking process in 4 SOLO (Structure of the Observed Learning Outcome) taxonomy levels; those are idiosyncratic or pre-structural, transitional or uni-structural, quantitative or multi-structural, and analytical or relational. Shaughnessy et al. (2005) categorize this statistical reasoning into 3 reasoning levels; those are additive, proportional and distributional, while Reading & Reid (2006) divide it into 4 levels, they are pre-structural, uni-structural, multi-structural and relational. The aforesaid studies have developed statistical reasoning model without further examining the statistic reasoning process.

From the interview with students of Mathematics Education Program of Study, it was obtained that the students feel they can understand statistics material when they are taught about it. Moreover, they obtained a better score in the midterm test and a final exam. However, most students fail in statistical reasoning. It can be seen when the students find difficulties in making a conclusion related to the graphics. Moreover, from the observation result, it was revealed that the materials which are delivered by the lectures have been visualized in graphicss presented in Microsoft Excel and SPSS. The result of this study is supported by the research conducted by Shaugnessy et al. (2009) in which the students were difficult in reasoning distribution and graphics representation from distribution. Friel, S. N., Curcio, F. R., & Bright, G. W. (2001) state that most books used by the students at school only demand students to be able to read the graphics. Even though they are asked to draw graphicss, the purpose of this task is only to learn how to draw graphicss from the numerical scale. Students think that in drawing a graphics, it must be based on the data value known.

Based on the explanation above, this research formulates the following problems:

1. Students believe that they are able in statistics when they are taught, they also assume that they obtain a good score in mid-term test and a final exam. Yet, they are still weak in statistical reasoning.
2. Students are difficult in statistical reasoning related to conclusion drawing particularly in statistics graphicss.
3. Students are still hard in using statistical reasoning, although the materials have been taught both verbally using excel or SPSS and verbal.

From the explanation above, the researcher formulated the research problem, "how is the students' statistical reasoning in statistics graphics representation?" Meanwhile, the purpose of this study is to describe students' statistical reasoning in statistics graphics representation.

RESEARCH METHOD

The subjects of this research were semester IV students of Mathematics Education Study Program at Muhammadiyah University Ponorogo, academic year 2016/2017 who have taken a basic statistics course. The reason was that these students have already taken a course related to descriptive statistics which discusses graphics representation and data distribution. The research method used is descriptive qualitative.

The data collection technique employed questions related to graphics representation done by the students. Here, the students were asked to do the essay test related to graphics representation. The assignment used in this research involved the comparison of battery life variation of three brands, which is battery presented in one statistics graphicss from histogram type. The instrument used in this research was adapted from Bakker & Gravemeijer (2004) in Ben-zvi & Garfield (2004). The students were required to decide one battery brand among the other batteries. It was to discover the information of students reasoning in data distribution.

Next, the students were divided into two groups, students with poor statistics ability and students with high statistics ability. This research was according to the basic statistics score in semester III. The students' reasoning of each group had been coded to be classified later by using thinking hierarchy suggested by Reading & Reid (2006) who divide it into four statistic reasoning levels; pre-structural, uni-structural, multi-structural, and relational. The researcher described each level based on students' reason, pre-structural category to the students who cannot give any reason, uni-structural category when the students focused only one key element from distribution, multi-cultural category when the students focused on more than key elements from distribution, relational category when the students were able to develop relational connection among various key elements of distribution. Besides, the researcher also carefully analyzed the reasons given by the students.

RESULT AND DISCUSSION

After completed the test, students were interviewed to clarify the answer. This interview result then was categorized into two groups, based on the students' statistics ability. It was students with low and high statistics ability. In addition, it was based on the final score of a basic statistics course. This interview was then analyzed.

Subjects analyzed from the student's group with low statistics ability were LK and LI. While subjects analyzed from students group with high statistics ability were IAA, AFR, and BTN. This analysis presents statistical reasoning ability of the low group students who were difficult in concluding data. Meanwhile, students' statistical reasoning ability from high group tend to employ informal

statistics reasoning. Besides, students with high statistics ability tend to employ multi-structural reasoning level. This table below presents students with high and low statistics reasoning ability.

Table 1. Students Reasoning with Low Statistics Ability Analysis

Subject Name	Ability	Provided Statistics Reasoning	Reasoning Level
LK	Low	Difficult to explain in words Mark A 120-130 Mark B 100->140 Mark C 100, 100, 100 etc	Pre-structural
LI	Low	Written in numbers Mark A 120, 130, 122, 125, 130, 120, 120, 120, 120, 120 Mark B 140, 120, 100, 160, 120, 100, 130, 110, 110, 160 Mark C 100, 100, 100, 100 etc Choosing C because of the stability	Pre-structural

LK is a student belongs to low statistics ability group. According to the answer provided by LK in solving question related to graphics representation, it can be seen that LK was difficult in generating statistical reasoning related to distribution. It can be read from the LK's answer as follows.

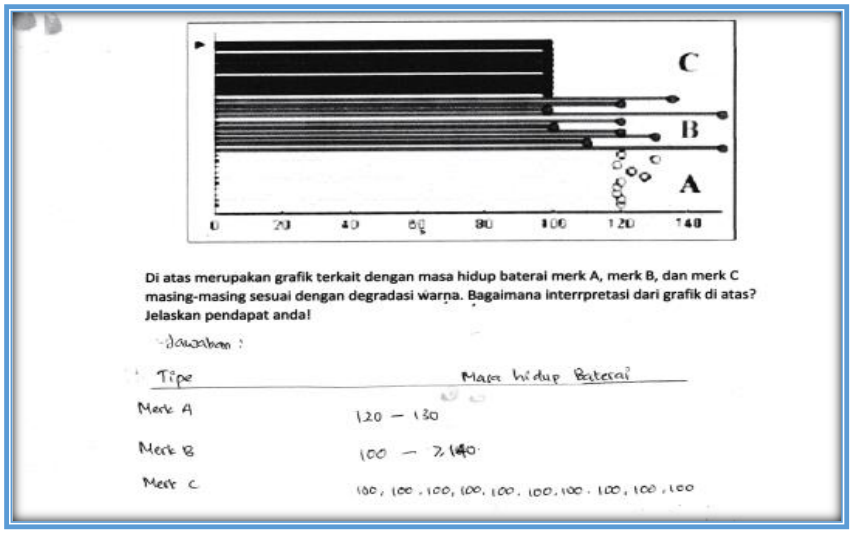

Figure 1. The Answer Provided by LK

Based on the answer provided by LK above, it can be seen that LK could not draw a conclusion from the data representation given, even though they were required to do so. It was because LK wrote data as a collection of the individual score, not as one score representing from the mentioned graphics. The sample of conversation with the subject is presented as follows:

R : How did you read these graphics?

LK : Well…like what I wrote, Ma'am

R : Then… what can you conclude from that graphics?

LK : I'm confused Ma'am… I can't make any conclusion.

R : Why?

LK : Well, I'm just confused (laughing)

According to the interview result above, it confirms that LK was difficult in providing statistical reasoning related to distribution.

LI is the subject belongs to low statistics ability group. According to the answer provided by LI, it can be seen that L1 seemed could draw a conclusion but L1 could not provide any statistical reasoning, as it describes in this sample of the interview below:

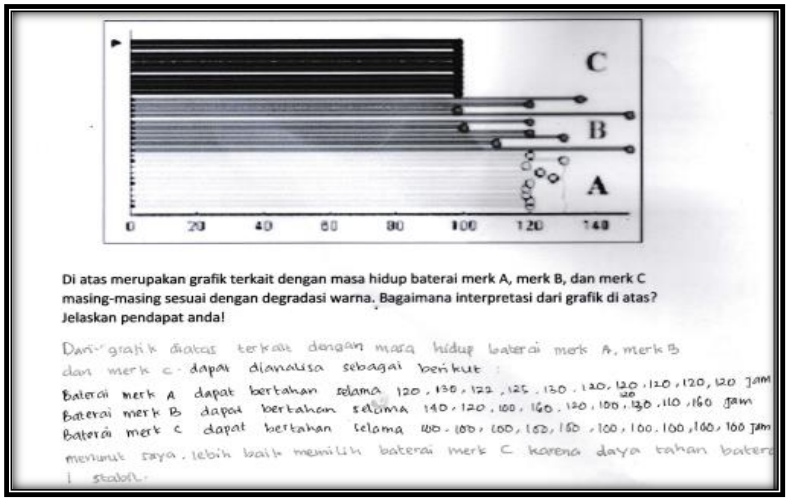

Figure 2. The Answer Provided by LI

According to the answer provided by LI above, it can be seen that L1 could draw a conclusion from data representation given. It was because LI could write the conclusion that the battery chosen is battery C. LI provided a reason which battery C life is more stable. From this answer, then the researcher conducted an interview to clarify the answers provided by LI. The sample of the conversation related to the answers of subject LI is as follows:

R : What does this 'stable' mean from your answer given?

LI : It's same, Ma'am.

R : What do you mean with 'same'?

LI : The battery C life is same, Ma'am. It is 100 hours

R : Could you give me another reason why did you choose battery C?

LI : No, Ma'am.

From the interview result, it can be concluded that LI could draw a conclusion but could not give any statistical reason. It was because the answer given by the student about the choice of battery C was based on the individual judgment from the data above.

Table 2. Students' Reasoning with Low Statistics Ability Analysis

Name of Subject	Ability	Provided Statistics Reasoning	Level
IAA	High	A is less stable B is much not stable C is stable Choosing B because the cumulative average of life is longer compared to others	Multi-structural (stable and average)
AFR	High	The life of battery A is almost the same. The life of battery B is much different compared to others. Battery A is more stable and long-lasting	Relational (relating between stability and endurance)

IAA is the subject in high statistics ability category. From the answer provided by IAA in Table 2 above, it can be revealed that IAA could make a conclusion from data representation related to distribution by providing informal statistical reasoning. IAA used terms stable and cumulative average. From this answer, then the researcher confirmed the answers provided by IAA by conducted interview as seen in the excerpt below.

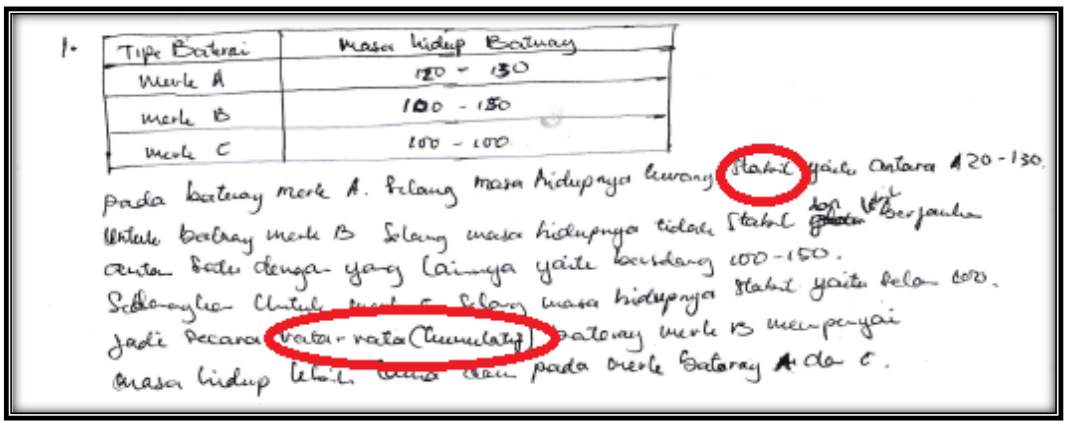

Figure 3. The Answer Provided by IAA

R : You said that battery A is less stable. How do you explain it?

IAA : Well… for battery A, there is the same life period and the different life period. Yet, the difference is not that far, therefore I said it is less stable. Meanwhile, battery B has no similarity at all and the difference is far, that is why I said it is not stable.

R : Where is that far difference?

IAA : Those dots, Ma'am (pointing dots at the edge of each line)

R : Now, what is the cumulative average?

IAA : The life period average of battery C is less than the life period of battery A, and less than the life period of battery B. So the average of battery A and B is not close, while battery C has the lowest life period average.

R : What do you mean with the closeness?

IAA : The value is not that different.

R : Ismail said about average… what is that average?

IAA : The total of all numbers divided by frequency (n/f). On the other words, a number that has almost the same value with the others (between high and low numbers). The point is the average (smiling)

R : And you also mentioned about cumulative average if I'm not mistaken?

IAA : Yes, that's right

R : How does it differ from the usual average?

IAA : Well, what I mean is not that cumulative average, but average or cumulative score (an index score)

R : Average value or cumulative value?

IAA : Yes, Ma'am… all values are sum up and divided by the total data

From the interview result, it can be noted that IAA could draw the conclusion and provide a statistical reason. IAA used informal statistical reasoning. It is recognized from the students' answer about the choice of battery B by considering stability and the cumulative average. Also, from the interview, IAA confirmed what was meant by stable is the variation. Yet, IAA regarded that variation as a number of different data. While the cumulative average is average from the value per battery. In addition, the subject chose battery A first because it was more stable than battery B, but at the end, the subject chose battery B due to its longer life average compared to battery A and C. IAA concluded without considering the relation of these two. Therefore, the researcher considered it in multi-structural level.

On the other hand, AFR is a subject that is classified as high statistics ability category. From the answer delivered by AFR in the following Figure 4, it can be seen that AFR could conclude the data representation related to distribution by giving informal statistical reasoning. AFR used terms of stable and relative average.

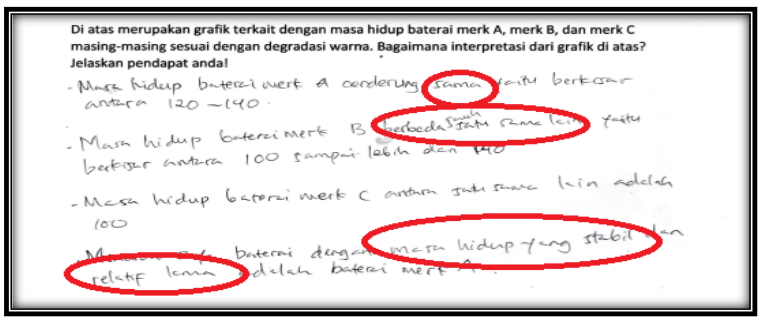
Di atas merupakan grafik terkait dengan masa hidup baterai merk A, merk B, dan merk C masing-masing sesuai dengan degradasi warna. Bagaimana interpretasi dari grafik di atas? Jelaskan pendapat anda!

Figure 4. The Answer Provided by AFR

From this answer, then the researcher confirmed the answers provided by AFR by conducted interview as seen in the excerpt below.

R : What does stable mean in Azizah's answer?

AFR : It means the life period is different, yet the difference is not that much

R : What kind of difference is it?

AFR : Well… battery A is 120 or more. Yet the difference is not that far.

R : Then what do you mean by its relative long on your answer?

AFR : What I mean by its relative long is achieving at least 120

R : You mean, you compared it to decide that minimum 120 is relative longer?

AFR : Yes, Ma'am. I compared it with battery B and C

R : What did you compare?

AFR : The time, Ma'am. So I take x-axis as time. The battery C life is only 100, then although there is battery B life 140, there is also the battery B life which is only 100. That is why I assumed that battery A has relative longer life which is 120 and 120 more.

From the interview above, subject AFR could write the conclusion and provide a statistical reason. AFR employed informal statistical reasoning. It can be seen from the subject's answer regarding the choice of battery A by considering the stability and its relative average. From the interview, it is found out that the stability meant by subject AFR was the variation. AFR, however, regarded the variation as the numbers of different data among others, similar to IAA. Meanwhile, what AFR meant with relative average was the average from the value of per battery. It was found out from the interview and the answer is given, that Subject AFR decided to choose battery A. Subject AFR made a conclusion with considering the relation between the variation and average. Thus, this subject belongs to relational reasoning level.

Reading & Reid (2006) argue that when children do statistical reasoning, there are four levels of the category; those are pre-structural, uni-structural, multi-structural, and relational. LI and LK are students with low statistics ability that are considered pre-structural category since they could not provide any reason when they were required to make conclusion related to data distribution. IAA who is a student with high statistic ability is considered multi-structural reasoning level since he could provide a reason although using informal terms, such as stable and cumulative average. Meanwhile, AFR is a student with high statistics ability also belongs to relational statistics reasoning level because he could associate one distribution to others. Similar with IAA, AFR also used informal terms though both of them have taken formal learning.

From the work results and interviews, LI and LK were not able to draw a conclusion. The subjects' answers were only based on the individual perception of the graphics. This is in line with the theory proposed by Konold & Higgins (2002) stating that the tendency of students in imagining data as a collection of individual perception is not an aggregate with particular characteristics.

IAA and AFR provided a reason related to the distribution leading to the variation measurement. Yet, these two subjects considered that the variation was a size showing the numbers of different data among others, not on the value showing the difference of the mean value. This explanation is congruent with the opinion of Kader & Perry (2007) who state that the probability seems to lead to the intuitive concept from variability unlikeability given by concerning the numbers of different data among others. The students did not consider variation as the value that is different from its mean.

AFR's reasoning level is in relational reasoning level. It is because AFR could conclude the graphics by underlying the relation between variation and average. As what it is stated by Konold & Pollatsek (2004) in Ben-zvi & Garfield (2004) that central tendency and distribution measurement are inseparable. In her answer, AFR used central tendency measurement by using informal terms which is relatively average and variation distribution measurement.

The use of informal student reasoning was a student effort in the search structure. As described by Miller-Jones (1991), Ralph & Anthony (1991) that informal reasoning is characterized as a search structure whereas formal reasoning is a structure of use. Furthermore, Perkins et al. (1991) explained that successful non-formal arguments that have a search structure must include all relevant aspects, not only from some aspects. This means that students who have informal relational reasoning are already successful in reasoning.

CONCLUSION

Based on the findings, some conclusions are obtained. It summed up that the semester IV students in Mathematics Education with low statistics ability are classified into the pre-structural level, meanwhile for students with high statistics ability are classified into the multi-structural and relational level. Students with high statistics ability could conclude data by statistical reasoning although they use informal terms. Students with multi-structural reasoning and relational level consider that variation as a measurement showing the numbers of different data among the others, not on how the different value of the average. Students in relational reasoning level drew a graphics conclusion by relating between central tendency measurement and the distribution measurement.

Supposedly, for the future researchers, it is important to carry out a research to discover why the students who have taken formal statistics learning still use informal statistics reasoning. Another research that is possible to conduct is how the students' reasoning process reaches relational reasoning level.

ACKNOWLEDGMENTS

The researcher would like to express the deepest gratitude to Dr. Abdur Rahman As'ari, M.Pd and Dr. Subanji, M.Pd as the advisors in Doctoral Program of Mathematics Education of the State University of Malang, for their enlightenment and guidance hence this article can be compiled.

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