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MATHEMATICAL MEANING IN MODELLING CONTEXT THROUGH THE ONTO-SEMIOTICS APPROACH

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Abstract

The main objective of this research will implement the onto-semiotics approach to analyse the conceptual of mathematical meaning in a modelling context corresponding to their use of the mathematical objects. Semiotics functions and mathematical object that emerged when solving mathematical modelling will be highlighted according to OSA. Students responses to modelling questions were used to classify the semiotics function that relates to the different mathematical objects.

Keywords: Mathematical Meaning, Modelling Context, Onto-Semiotics Approach

Some mathematical problems required students to have many other abilities than mathematical concepts. For example, the mathematical problem in text forms enforced students to transfer text into a mathematical concept that is called mathematical modelling. Mathematical modelling is an important process in mathematical classroom learning (Rellensmann, Schukajlow, & Leopold, 2017). Students need to apply several abilities to make mathematical modelling.

There are three phases in mathematical modelling, first, students need to understand the text carefully and try to imagine a mathematical mental model prior to making a mathematical concept(Ero-Tolliver, Lucas, & Schauble, 2013). Secondly, students need to inference an important information and eliminate some unimportant information to make a focus about the problem is given. Thirdly the last phases, students have to draw a mathematical modelling based on information given. The three phases are ideal phases which need to be taken by students in performing modelling. Students use many symbol and representation during the mathematical practices and solving problems. (Leiss, Schukajlow, Blum, Messner, & Pekrun, 2010; Rellensmann et al., 2017).

Many researchers have been currently focussing on modelling problems, however, there is only limited research who has been a focus on analysis the mathematical meaning. We are going to use the onto-semiotics approaches to analyze the meaning of a mathematical symbol and representation that is used by students in solving modelling problems. We use (Garderen, 2006; Juan D Godino & Batanero, 1997; Mentzer, Huffman, & Thayer, 2014) Onto-semiotics approach to analyzing the meaning of symbol based on students work.

Mathematical Meaning

The topic of Mathematical modelling on educational research is really extensive, however, only several studies are concerned about mathematical modelling (Frejd & Bergsten, 2016). In modelling activity, students produce many representations, symbol, mathematical notations, semiotics functions during the entire process of modelling activity. They use symbols, numbers, lines, and other semiotics function which underlying some partial correlations in the mathematical context that associated with the activity of solving the modelling problems.

Any representations, symbols produced by students have a meaning that represents students' understanding both text comprehension and mathematical conceptual(Schukajlow, Kolter, & Blum, 2015; Van Meter & Garner, 2005). Many mathematical notations clearly represent the meaning of complexity in mathematical modelling context, the holistic meaning of mathematical notation used in mathematical practice system represent the different expression that closely related to other semiotic functions and mathematical conceptual understanding (Montiel, Wilhelmi, Vidakovic, & Elstak, 2009). The holistic meaning arose from several coordinations of fragmentary meaning that closely associated with mathematical semiotics functions (Malaspina, Wilhelmi, & Barcelona, 2008). The meaning of mathematical notations combined with a contextual understanding of real situations has produced a complex comprehension about modelling problems.

We utilize the onto-semiotics primary entities to make understanding of mathematical meaning produced by students during the process of modelling activities. Mathematical meaning will be explored through six primary entities which are (Font, Godino, & D'Amore, 2007; Juan D Godino & Batanero, 1997) (1) language (terms, expressions, notations, etc); (2) situations (problems or intra-mathematical applications, etc.); (3) subjects' actions when solving mathematical tasks; (4) concepts, given by their definitions or descriptions (number, symbols, mean, function, etc.); (5) properties or attributes, which usually are given as statements or propositions; and, finally, (6) arguments used to validate and explain the propositions.

Modelling Context

Mathematical modelling currently has been a focus for mathematics education (Rellensmann et al., 2017) because mathematical modelling encourages the mathematical learning process into daily life activity that empower students not only to improve their mathematical problem-solving ability but also boost others ability that closely associated with their required life skills. Mathematics modelling activity encourages students to generate a drawing that represents their understanding of the problem, conceptual mathematics comprehension, and students acquisition of words (Van Meter, 2001). Mathematical modelling enforced students not only to master the mathematical competence but also insisted to foster others abilities and reasoning in understanding a real problem context.

Some drawings generated by students can be useful for teachers to explore their meaning toward modelling context(García & Bosch, 2006; Kaiser, Blomhøj, & Sriraman, 2006; Tan & Ang, 2016). For example, many students are failed to transfer the problem in text form into a mathematical concept, so teacher need to explore why some students get fails in drawing pictures, what factors that influenced them to fail. A teacher who believed the modelling as an important aspect in learning mathematics will use mathematical modelling context in their instruction (Tan & Ang, 2016).

Implementing modelling context into the learning process need to be developed from a different context (Frejd & Bergsten, 2016). This is mainly because modelling context is extensive which include the relationship between mathematics and workplace, mathematics and daily life, mathematics and professional life, and many others. For example, the relationship between mathematics and workplace has been generally focussing on how to implement mathematical concept into the workplace.

Onto-Semiotics Approach

Our theoretical framework will be limited to mathematical objects that are used by students. Students use many symbol and representation in mathematical practices, they use them to communicate, solve, explain, and argue towards problems(Stender & Kaiser, 2015). They represent the mathematical objects that have been currently a focus for mathematics educational research. The mathematical object will be limited to "any symbols and representations that are used to communicate, clarify, point out, draw during the transition process from real-life problems to mathematical model". In this research, mathematical objects that appear during mathematical practices will be analyzed with the onto-semiotics approach.

There are six primary entities in the onto-semiotic approach namely (Font et al., 2007; Juan D Godino & Batanero, 1997) (1) language ; (2) situations; (3) subjects' actions when solving mathematical tasks; (4) concepts, given by their definitions or descriptions; (5) properties or attributes, such as statements; and, (6) arguments used to validate and explain the propositions. Language used in mathematical learning closely related to mathematical conceptual. Students use argument to strengthen their answers by using the mathematical definitions.

Our analysis using onto-semiotics approach will be a focus to mathematical objects. During the process of analysis, we will consider the following dual dimension (1) personal/institutional; (2) ostensive / non-ostensive. (3) example / type; (4) elemental / systemic; and (5) expression / content(Juan D Godino & Batanero, 1997; Juan Diaz Godino, Batanero, & Font, 2007). This analysis also will be considered the effect of students' previous knowledge, students life experiences, students intuition, students mental imagery on how they use these to comprehend and point out the problems before transferring the real-life problems into a mathematical model.

The onto-semiotic approach suggested five level of analysis for mathematical practices and instruction processes (Juan D Godino & Batanero, 1997; Juan D Godino, Granada, & Vicenç Font, 2006), the fist analysis is analysis of types of problems and systems of practices. The first level of analysis allowed us to explore any types of mathematical problems. While the second analysis is Elaboration of configurations of mathematical objects and processes, provide us with opportunity to identify any object used either in mathematical practice or solving problem. The thirdly is the analysis of didactical trajectories and interactions. The Fourthly is about the identification of systems of norms and meta-norms. Finally, was about the evaluation of the didactical suitability of study processes. The authors will focus on the first level analysis and sometimes also will collaboratively touch the second level as well for specific analysis.

METHODS

In this study, the researchers employed a qualitative method for analysing the meaning of semiotics representations used by students in group discussion. Our research used the proposed methodology for a study on an alternative theoretical framework for mathematics educations, which suggest this study to select particular problems or specific cases and analyse this problem under the theoretical perspective. Pythagoras Theorem is a part of the mathematical subject for junior high school students. Our tasks would propose students not only to solve problems but also to draw two different images; a situational drawing and a mathematical drawing (Rellensmann et al., 2017). A Situational image encourages students to draw a real situational about problems as specific as possible while a mathematical drawing encourages students to draw a conceptual framework about problems. The following task (Blum & Borromeo, 2009; Rellensmann et al., 2017; Schukajlow, Krug, & Rakoczy, 2015) encourage students to make.

Question 1

Malang government currently give a new fire engine with a turn-ladder to the officials. The cage at the end of the ladder will be use to rescue people from the highest placed. The official rules explained that while releasing people, the truck has to maintain a distance of at least 12 meters from the burning building.

Technical data of the engine:

Construction year : 2017
Power : 240 kw (279 HP)
Cubic capacity : 8,980 cm³
Dimensions of engine : length 10 m width 2.5 m height 3.19 m
Dimensions of ladder : length 30 m
Weight of unloaded engine : 17,240 kg
Total weight : 19,050 kg

From what maximal height can the Malang official can rescue people with his truck fire. You should complete the answers with a situational drawing, a mathematical drawing, and the appropriate solution for this accident.

Figure1. The question

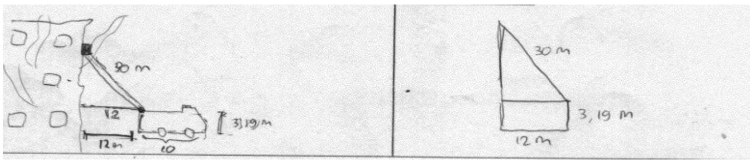
In this study, all students were first given a problem which proposed students to make two different pictures and to solve the problems. The resemblance students' works were collected and categorized by researchers. The eight students chosen will be interviewed, in a group of four. The activities in group will be recorded. Each session will take approximately forty minutes. The students should explain verbally their answer individually and a group discussion were promoted when another student was an inevitable response to their friends' work. We will exclusively analyse the first problems.

RESULTS AND DISCUSSION

The conceptual frameworks of onto-semiotics approach will closely attention into the primary entities and the classification of mathematical objects. There are eight students will be interviewed in two different groups. S1, S2, S3, and S4 will represent subjects in the first group while S5, S6, S7, and S8 will represent subjects in the second group. Each session will be interviewed at the same time but each group will be placed in different rooms to avoid the interruptions and noise.

The main objective of the first problems is the fact that students tend to use the usual numbers that are used in Pythagoras theorem which is not considered in fire engine problems context, in fact, the maximum height of ladder fire engine was asked, because there are only hypotenuse sides and wide sides in a context of fire engine case. Language is seen as the primary entity. For example, S4 personally explained and believed that 30 meters are the results both area and perimeter of a triangle that we need to discover, he did not use 30 meters as hypotenuse side. He only uses 12 meters as the side of the triangle, then he refers the Pythagoras theorem that associated with 12. He then finds 13, and 5. From S4 explanation, we can say that S4 does not understand the problems in fire engine context, while he has an understanding about Pythagoras theorem. Although he fails to use the Pythagoras theorem in fire engine context. Her understanding of problems is merely low. Her quotation and argument are a specifically personal object, although her group answer is correct.

Expected Answer

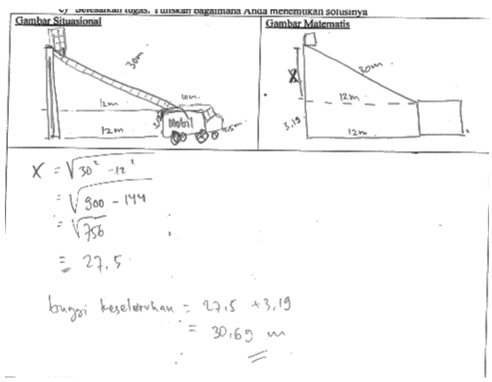


$x = \sqrt{30^2 - 12^2} = \sqrt{900 - 144} = \sqrt{756}$

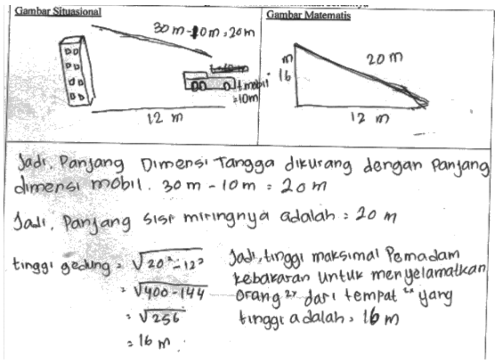
$x = 27.49\ meter,$ the value of x will be added with the height of fire engine car 3.19 meter

The maximum height that Malang fire official can help people is approximately about 27.49 meter + 3.19 meter= 31.68 meters.

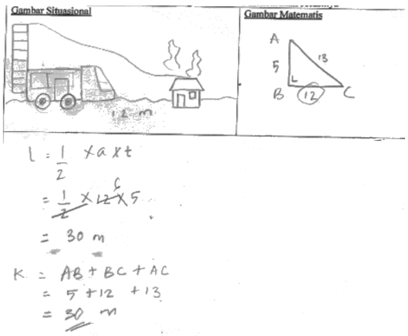
Answer From S1



Answer From S4



Answer from S2



Answer from S3

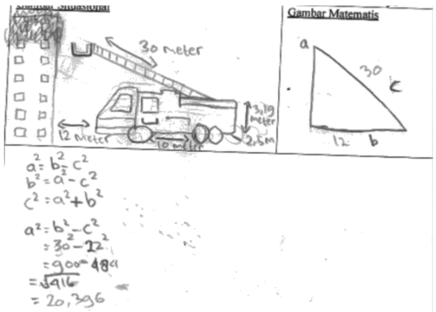


Figure 2. Expected answer and students answer

From the above figure, we can overview that S4 has given his explanation "the dimensional length of ladder minus the dimensional length of the fire engine, 30 meters -10 meters is equal to 20 meters, so, the hypotenuse is 20 meters", using his personal mental construction about the real situation, and Pythagoras theorem which underlying, S4 had produced a situational drawing, which largely relies on his personal mental perspective as well as a mathematical drawing. On the other hand, S1 produced the different a situational drawing and mathematical drawing by saying "The dimensional ladder of fire engine is 30 meters" using his personal perception about the real situation, and mathematical concept in its underlying, S1 produced a mathematical drawing and a situational drawing more accurate than S4, while S3 and S2 used his experiences to imagine the real situation about this problem. After S1 presented his answer to group discussion, the understanding S2, S3, and S4 about the problem real situation are more comprehensive.

The series of development is determined by the institutional requirements of (1) determining the product of situational drawing which is corresponding to the real situation problem, (2) assuring the product of mathematical drawing is represented both the situational drawing and the real situation context, (3) taking into account which side is represented the hypotenuse prior to apply the Pythagoras theorem, (4) connecting the mathematical answers with the real situation context. By doing this sequential works, students will be able to construct a mathematical model. As a result, students will not only become more fluent in applying the Pythagoras theorem but also they maximized the opportunity to solve the mathematical modelling problems.

The onto-semiotics approach has seen the concept (definition) from different mathematical context. The Pythagoras theorem was initially presented in junior high school level, usually after learning algebraic system and operation. When a teacher has given the underlying structural definition of Pythagoras theorems ("the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides"), what remains in students' mind is the area and circumference of the triangle. Even the Pythagoras theorem is quite simple, some junior high school students failed to apply this theorem mainly because students usually fail to understand the problem so that they cannot make a mathematical drawing as well. Furthermore, mathematical textbooks in school rarely used modelling problems for Pythagoras theorem, they prefer to use many right triangles that placed either vertical or horizontal forms. Students will be requested to discover the length of a side, where two others sides have given.

In this task, it can be clearly explained that "phenomenon" comes from the Pythagoras theorem system that in the form of "$a^2=c^2- b^2$" or $b^2=c^2- a^2$". In this task, there is some unnecessary information that students need to avoid, for example, a data technical engine has given a lot of unnecessary information. Students need to see an important information and unnecessary information. This dual dimension example/type and expression/content need to be avoided by students so that they can select an useful information correctly.

CONCLUSION

It is fundamental to understand the real problems and visualize a real situation in both a mental construction process and drawing context. To construct a mathematical drawing and situational drawing, students need to understand the problems before communicating our ideas into mathematical concepts. To communicate the ideas, several symbols, and semiotics functions has played an important aspect in understanding the problems. Onto-semiotics approach has seen that notations and arguments play an important rules in the modelling process.

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