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THINKING INTERACTION OF STUDENT IN SOLVING OPEN- ENDED PROBLEMS

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Abstract

The purpose of this research is to describe the process of thinking interaction of students in solving the open-ended problem. The thinking interaction of students' in this study is defined as an activity of exchanging thoughts between students one with other students through communication to produce the suitability of the thinking process between them through the resolution of open-ended problems. This study was conducted on a group of junior high school students with heterogeneous capabilities, consisting of 5 students, and have taken the material to build space. The data of thought interaction activity is obtained from discussion result in solving open-ended problem analyzed by information processing theory. Students' thinking interaction activities begin from one subject person who gives statement and understanding about the problem provided. Activity mutual ideas, complement or give feedback to other subject's comments, ask questions that have not been understood, or give an explanation in deciding the answer to be agreed. Subsequent activities decide the answer orally to be written by each subject. All subjects poured the agreed result on the answer sheet, but there were still doubts from the two subjects on the answer, so the subject looked back at the answers from other subjects. Some subjects continue to write answers and in the end, each subject has a form of answer in accordance with their respective understanding.

Keywords: Interaction thinking, problem, open-ended

The main target of the achievement of the learning process is the change of knowledge and the increasing of students' experience about the science they learn. This change of student knowledge must be assisted through interactive activities with him/herself, other students, teachers, and the environment. Students must construct knowledge from the environment, interact with experiences and objects encountered, and perform abstractions, either simply or reflectively. In line with NCTM (2014:9) that learning mathematics is an active process, in which each student builds his own mathematical knowledge from personal experience, coupled with feedback from colleagues, teachers and other adults. Conducting interaction activities in constructing knowledge is one way to be done by students. Interacting requires the involvement of others or the environment in order to get maximum results. Johnson & Smith, (2014) suggests the involvement of group members in discussions can contribute maximally to each member of the group and facilitate the achievement of the shared goals.

Student involvement actively in group discussion activities is a way that can help students to construct their own knowledge. To support this, the teacher should provide the widest opportunity to his students in the discussion activities. This is in line with the dissemination by Stockero & Zoest, (2013) and Santagata, Zannoni, & Stigler, (2007). Stockero, et al, (2013) suggested to teachers that be patient not to notify in advance when there are problems faced by students in the implementation of learning and expected to be given the opportunity to students to improve and find the solution first. Santagata, et al. (2007) suggests giving students the opportunity to solve math problems until they get into trouble and need help from the teacher. The presence of teachers is necessary but requires the right way and time. Implementation of learning with group discussion can reduce the participation of teachers during the learning process. This is in line with his opinion Kidron, et al. (2008) that significant mathematics learning cannot be achieved if the student's work depends too much on the teacher. Therefore, teachers should understand and know the interaction of students in the group.

Kumpulainen & Wray, (2002: 36-39) devised three dimensions as a method for analyzing student interactions in peer group discussions. Three dimensions as a method for analyzing such interactions are cognitive processes, social processes, and language functions. Furthermore, Bishop, (2014) studied student interaction activities in groups to position themselves for each other during work. In addition, Routledge & Radford, (2011) and Radford, (2011) have produced several diverse ways to conceptualize interactions in the group. Radford, (2011) discusses the development of methodologies and theoretical tools for investigating interactions, the role of teachers, interaction and dialogue in the classroom, and the relationship between thinking and speaking. Radford, (2011) studied the layout of the room to support the learning process so that student conversations with students are directly guided by the teacher, forming small, directional discussion groups and individual problem solving supported by computers in collaborative learning. Methods of positioning students in groups greatly helped them to maximize group members actively construct knowledge. Member involvement in discussions can maximize individual contributions and easily achieve common goals (Nilsson & Ryve, 2010).

Research related to group discussions to challenge, debate ideas, give opportunities to convey ideas, unite perceptions, or practice to communicate effectively performed by (Weber, Maher, & Powell, 2008; Nilsson, et al, 2010). Weber et al. (2008) suggest that students often challenge arguments from their peers who present the material, so these challenges can invite students directly to understand the principles of mathematics or indirectly use as a basis for understanding mathematics. Will be maximum discussion results if different speeches from the other person can generate responses that are consistent with what the speaker is expecting. The occurrence of mutual responses to the material being discussed will occur if the issues to be discussed sometimes some members of the group have not understood well. Understanding certain materials by each student are different. Some really understand some are still missing, some are still wrong, and some even have not understood them at all. This happens because a person's ability to observe, analyze or the ability to think about it is limited.

Many researchers have investigated students' ability to think in this regard to mathematical thinking (Peterson & Leatham, 2009; Bishop, Lamb, Philipp, & Whitacre, 2014). Peterson & Leatham, (2009) examines the setting of class discussions based on students thinking mathematics, explaining learning steps by using students 'mathematical thinking, and identifying some student barriers from listening, understanding, and effects using students' mathematical thinking. Bishop, et al. (2014) examines the mathematical thinking of children and the idea of cognitive impediments. Further

Stockero & Zoest, (2013) suggests that at key moments teachers should provide key instruction to build students' minds about mathematics. Research on students thinking mathematics is very varied and diverse. His research has photographed him from the learning process, from interaction activities by working on specially designed questions or by giving difficult questions.

The learning process involves the interaction between the teacher and the student or the student with the student will involve the student's thought processes. Student thinking activities in mathematics always vary and differ from one student to another. Investigating the variation of students' thinking on mathematics has been studied by researchers (Leatham, Peterson, Stockero, Zoest, 2015; Zoest, 2017). Leatham et al (2015) devised a framework concept for analyzing all examples of mathematical thinking of students that took place during classroom instruction. Van Zoest et al. (2017) investigated examples of mathematical thinking of students during interactions identified as having high potential. Forms of thought (for example, questions vs. declarative statements), whether the thought is based on previous or generated work at the time, the accuracy of thought, and the type of thought. The results of this study provide an idea that students' minds about mathematics vary widely. This student's mind can be mixed in the interaction of group discussion between one student and the other. Interaction activities like this besides helping to unify students' understanding of certain materials, can also help in solving problems.

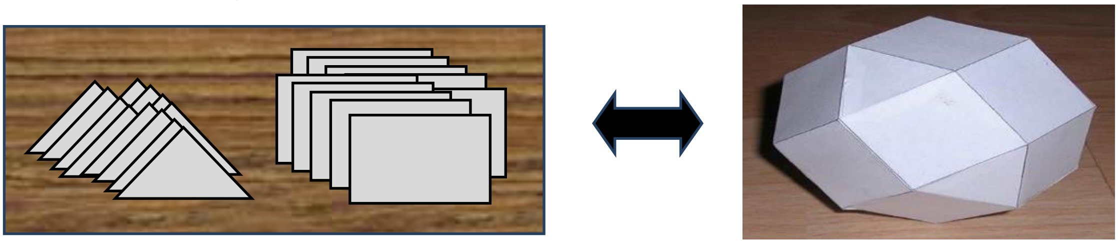
Some research on patterns of student interaction with problem-solving done in clusters (Barron, 2003; Watson & Chick, 2001; Lai & White, 2014). Barron, (2003) Investigating collaborative interactions can influence problem-solving results and find that quality Interaction has implications of learning outcomes Watson, et al, (2001) identifies cognitive, social, and other factors affecting problem-solving results achieved during collaboration Lai & White, (2014) suggests that problem solving when undertaken in group discussion activities will Positive impact if coordinated well and negative impact if not coordinated properly problem-solving done in group will help accelerate finding problem solving process Collaboration activity often seen by learners to support idea each other by offering missing parts from Information needed to break down It's a matter of (Francisco & Maher, 2005). Furthermore (Mueller, Yankelewitz, & Maher, 2012) convey that students can work together and respond to their peers to jointly build ideas, but in other cases, one student can present a fully formed idea for others to expand or modify, as well as doubt the idea of his friends. Student activities to solve problems together will bring them rich in practice and knowledge.

One way that helps students will have rich experience and knowledge is that students are given an open-ended problem. Capraro, Capraro, & Cifarelli, (2007) explains that open-ended problems provide a free and supportive learning environment for students to develop and express their mathematical understanding. Sawada, (1997) mentions five advantages of open-ended problems. First, students are more active and free to express their ideas in the learning process. Second, students have greater opportunities and skills to use mathematical knowledge. Third, all students can answer the problem in their own way. Fourth, providing students with rational opportunities. Fifth, students are offered to feel the fulfillment of discovery and approval from other students. Kwon, O. N., Park, J. H., & Park, (2006) revealed that being given an open-ended problem can give students a sense of accomplishment and even for students with less ability to set their solutions based on their abilities. Therefore, with the interaction activities of one student with another in solving the open-ended problem will add to their experiences. The purpose of this research is to describe the process of thinking interaction of students in solving an open-ended problem. This study will focus more on students' interaction activities in small groups.

METHOD

This research is a qualitative research with descriptive exploration research type. Researchers chose 5 subjects consisting of 2 men and 3 women who have heterogeneous capabilities. Data retrieval was done by researchers recording the subject discussion activities that are solving the problem of math is about open-ended. The results of the videotape were transcribed to facilitate the analysis. The video recordings were analyzed and presented qualitatively based on Information Processing theory. The following are the instruments used in this study:

Look at the following pictures!



(i) (ii)

Information:
Figure (i) consists of several parts of the Same and Square Triangle and the Picture (ii) is formed from the arrangement of some parts of the triangle and square.

Question:
1. If 100 parts of triangles and 60 square sections are given, determine the possibility of building blocks of images, cubes, prisms, and pyramid!
2. Calculate the surface area of each of these beams, cubes, prisms, and pyramids that may be formed!

Figure 1. Instruments

RESULT AND DISCUSSION

Result

The results of this study were analyzed by employing the Information Processing Theory. Information processing theory according to Gagne (1985) has three main components, they are information storage components, components of cognitive processes, and metacognitive components. The information storage component consists of sensory memory, working memory, long-term memory. The components of cognitive processes consist of attention, perception, rehearsal, encoding, and retrieval (Hitipeuw, 2009). Of the three main components in the theory of information processing, in this study described into sensory registers, attention, perception, short-term memory (STM), long-term memory (LTM), rehearsal, retrieval, and encoding components in the theory of information processing will be used to analyze the interaction activities of students thinking in solving the problem of open-ended. The following figure 1 is the structure of open-ended problems used in this research.

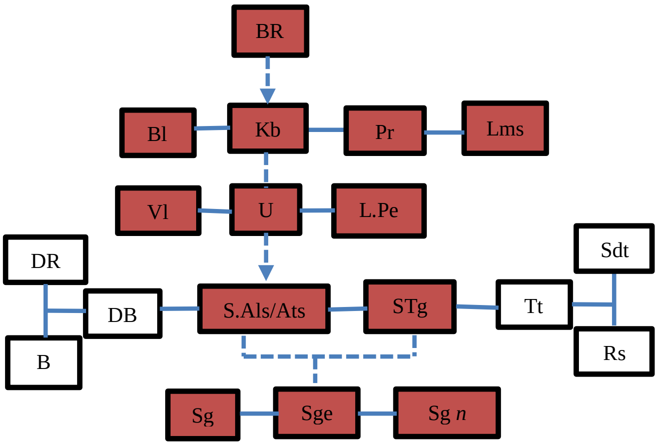


Figure 2. Problem Structure

Information:

	: Problems are discussed	BR	: Geometry	Un	: Element
	: Problems ignored	Blk	: Beam	DR	: Diagonal of Space
	: Includes	Kbs	: Cube	BD	: Diagonal Field
	: Relationship	Prm	: Prism	DB	: Diagonal Fields
	: Part of	Lms	: Pyramid	Ttk	: Point
S.Als/Ats	: Alas / Top Side	Vlm	: Volume	Sdt	: Corner
Stg	: Upright Side	Sge	: Rectangular	Rsk	: Lateral
Sgt	: Triangle	Sg n	: *n* side		

The activity of solving the open-ended problem by the subject is done by discussing in one small group. The subjects who held the discussion there are 5 people who will be symbolized S1, S2, S3, S4, and S5. The activity of solving the open-ended problem begins with all subjects understand the problem (stimulus) given. All subjects looked at and observed (sensory registers) of the given problem. Subjects focus (attention) their minds to understand the problem, then communicate (retrieval) the results of his understanding of other subjects.

Subjects who first started to reveal the results of his understanding is S1. S1 began to reveal about a known problem there were 100 images of equilateral triangles and 60 square images. Subject S1 has looked (sensory register) and understand (attention) of the given problem. Knowledge of equilateral and square triangles required by (short-term memory) stored on (long memory) is communicated (retrieval) to their friends. Knowledge of equilateral triangles and squares as materials to answer the question of how many triangles and squares are required to construct cubes, beams, prisms, and pyramids (long-term memory). The S1 statement is supported by S2 which directly shows the illustration of the problem already provided. S1 subject statements about triangles and squares for compiling cubes, beams, prisms, and limas are then supported by S2 as the following video transcripts:

S1 : this is ... there are 100 triangles and 60 rectangles to arrange cubes, beams, prisms, and pyramids (understand the problem while communicating with his friends).

S2 : this is the picture . .... (responding from S1 by illustrating from the provided image on the problem of some triangles and square to arrange as in the picture (ii).

Based on the dialog between S1 and S2 it can be said that the stimulus into the sensory register. In sensory registers, this is the place of the most outside information storage that directly captures or deal with the stimulus. Stimulus through the sense of sight and hearing. What is seen will be recorded as it is and has not been interpreted personally. Once the stimulus into the sensory register, then emerged the attention on S1 and S2, in particular, are expressed through the dialogue. Attention activities by S1 and S2 are not quite as expressed as the dialogue, a subsequent dialogue occurs from all subjects to discuss the most appropriate answer. Here's their advanced dialogue.

S1 : yes then again (responded by conical settling about the cube)

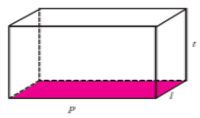
S3 : yes, but the triangle and square are for the cube, the beam, the prism, the pyramids.

This subject dialogue illustrates that there is a mutual reinforcement of attention. The S1 subject tries to direct the answer, but the other subject is still hesitant and asks to confirm again answer from S1. It still seems that the perception of each subject is not the same. So there is a dialogue like a transcript as follows.

S1 : yes, we spend triangles and squares to build cubes, beams, prisms, pyramids

S2 : if 20 square to construct the beam how?

S1 : four (pedestal and top), four (front and back), and 2 (left and right). (Counting the side of the block, if the square is used to arrange the side of the beam). Here's the drawing used as a comparison material to calculate all the sides of the following blocks:



S4 : yes right (while also counting the sides of the beam)

S5 : yes (come drawing)

These subjects mutually reinforce perception by summing up a few squares that can be used to construct blocks. Subsequently, the subjects realize the perception by drawing up space in the form of blocks in short-term memory. This is because when the stimulus has been perceived by someone means the person simultaneously pass the information that has been interpreted to short-term memory (STM). Gagne (1985) states a similar thing to this incident when one gives the perception it will be transformed into input into short-term memory. Subject-subjects (S1, S2, and S3) perform rehearsals in the STM against what is already about the square used to construct blocks of long-term memory

(LTM). The next dialogue takes place between all subjects to complete from their respective answers. Here's the transcript of their dialogue.

S3 : one, two, ..., ten (calculate the square that can be used to arrange the beam image)

S1 : this is ... (recalculate how many rectangles it takes to construct blocks)

S2 : oh so (while resuming the picture)

S5 : like this picture (while showing the picture)

S1 : yes like that, but given dashed (drawing dashed line)

S2 : (while drawing and sometimes while looking at the answer from S1, naming how to give different beam name with S1)

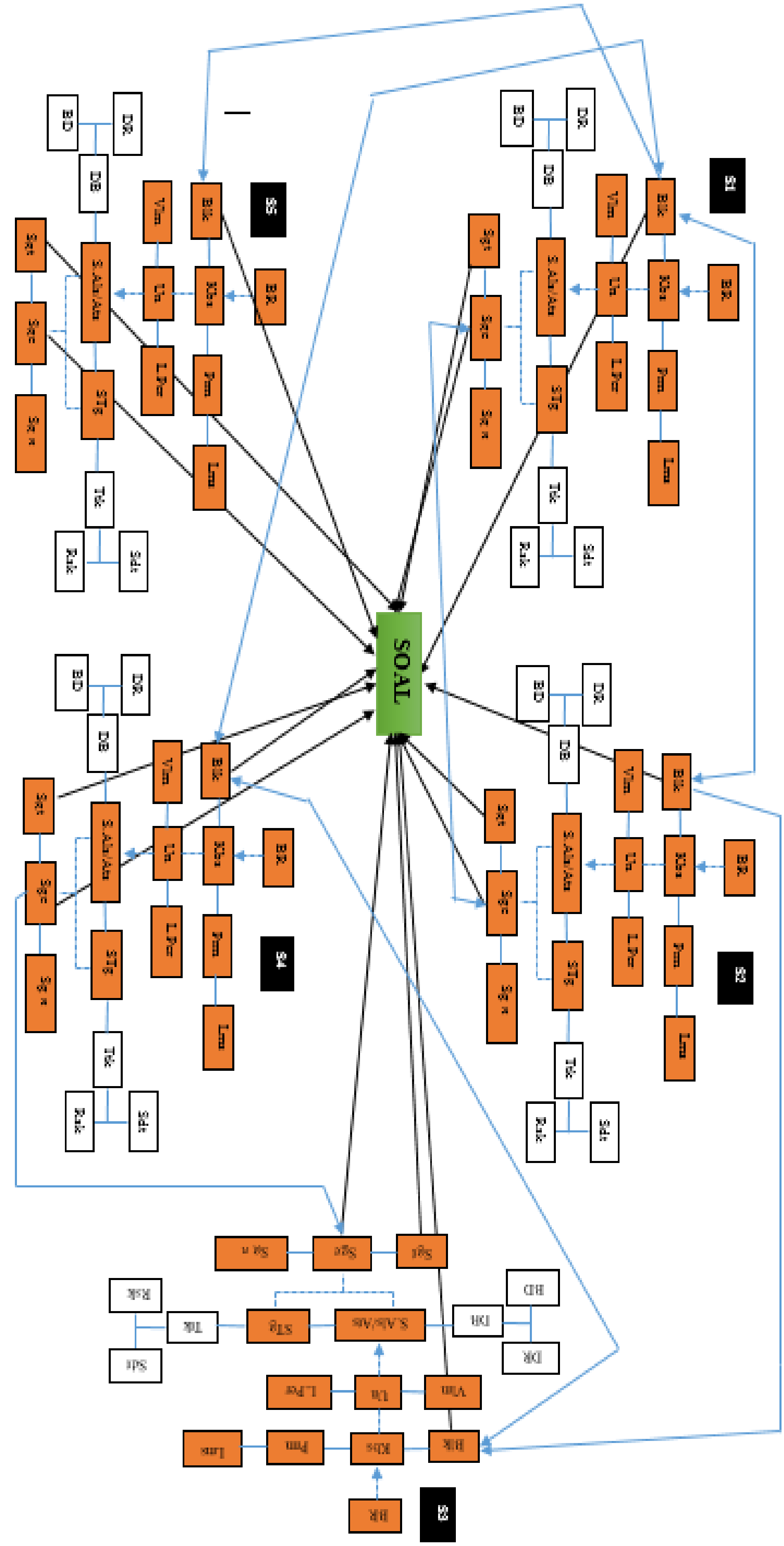
S3 : this is yes, (while writing numbers 1, 2, ..., 10 on the beam that shows the number of squares)

S4 : (draw a block by releasing 1, 2, ..., 10 on a beam showing the number of squares)

S5 : like this yes, (he wrote the letters ABIJ, IJDM, ... which is in the image Beams)

At the end of this dialog, all subjects already have one perception of what the answer will be written. Perception is a person's interpretation of the stimulus received. Perception is to transform the stimulus into an image of the object. Subject S5, S3, S2, S4, and S1 immediately draw an agreed answer. This means that S5, S3, and S2 realize the existing perception on short-term memory (STM). These subjects rehearsal in STM on what has been given and retrieval (retrieved) the stored knowledge of drawing blocks with all its sides square in long-term memory (LTM). Here are the answers of all the subjects to draw the blocks in their own way.

Based on the results of the answers to these subjects, there are different ways of answering although the objectives are the same. This illustrates that their knowledge of drawing up different spaces that exist in long-term of memory. Their retrieval knowledge activities on drawing blocks in LTM are different, in addition to how they represent different symbols and images. This is seen in the codes from the results of student answers above. To facilitate the flow of the process of thinking interaction of students in solving the problem of open-ended can be seen in Figure 2. Problem-solving activities as described previously only in one answer only. For other answers already answered by these subjects, but the way they do is almost the same as this one has been described. Therefore, the researchers only present this data as an analysis material to see the interaction activities of students' thinking.



Picture 2. Interaction Process Thinking The subject in discussion solved the problem

Information

: Includes	: Interaksi searah	Sdt : Corner	Rsk : Lateral	Tik : Point	STg : Upright Side	Sg *n* : Segi *n*	
: Relationship	: Interaksi dua arah	Blk : Beam	Vlm : Volume	DR : Diagonal Ruang	S. Als/Ats : Alas / Top Side	: Problems are discussed	
: Part of	BR : Geometry	Kbs : Cube	Un : Unsure	BD : Field Diagonal	Sgt : Triangle	: Problems ignored	
		Lms : Pyramid	L.Per : Surface area	DB : Diagonal Fields	Sge : Rectangular		

Discussion

The interesting result that will be discussed in this research is the interaction activity of thinking the subject in solving the problem begins with mutually reinforce the understanding of the problems of the problems that have been provided. Subjects give each other a perception of the results of their understanding of the given problem. Subjects with one another give a statement about the answers to be written together. Gresalfi, Martin, Hand, & Greeno, (2009) stated that the negotiation process is needed to reinforce the answers of each other's way of thinking. The activity of giving each other answers is not enough with verbal only. Apparently, there are still subjects who are not sure of the answer to the agreement orally and still see the answer from his friend. Webb, (1989) suggests that the opinions expressed in collaborative activities should be acceptable and appropriate from those receiving assistance. This activity will be preceded by S1 explaining the intent and purpose of the problem in the provided matter.

All subjects understand the given problem and the information goes into the sensory register, here S1 begins by identifying the problem after the information through the process of attention and utilizing the information on the matter to be communicated to all his friends. Other subjects are still there who are confused and wondering their friend. The individual's ability to identify the given problem will be influenced by the experience and knowledge of the individual itself. The experience and knowledge stored in LTM will affect the individual perception of the coming stimulus (Gagne, 1985). Experience will provide the form and structure for the object that the senses are captured, while the knowledge will give meaning to the object that the individual captures, and finally the individual components will play a role in determining the availability of answers in the form of attitude and behavior of the individual to the existing object. Gagne, also explains that the perception of a person depends on how widely the person has studied the stimulus in previous learning.

Subjects who still do not have the knowledge and experience stored in LTM will find it difficult to explain from the problems that exist in the working memory. Ryve, Nilsson, & Pettersson, (2013) argues that in high-ability group work activities status and social rules greatly affect interaction. Here will be the interaction of thinking subjects who already have previous knowledge and experience. Subjects who already have will explain to their friends. Vice versa, for subjects who do not have it, will receive the knowledge and experience. From the interaction between students, it is seen that the interaction that occurs in accordance with the view Vygotsky on scaffolding, which is one child can be more guiding the other child in passing ZPD (Zone of Proximal Development). Students who do not have knowledge of the issues to be resolved will make some misidentification of the assumptions given when giving their opinions, resulting in a mistaken perception and error in the answer. The incorrect identification process is due to the misconceptions and misconceptions stored in LTM. As a result, subjects with less understanding and experience have a false perception of the stimulus and difficulty in choosing a strategy or determining the initial step of the answer. This is in line with the results of (Weber, 2002; Barnard, 2015) studies which suggest that the initial cause of student failure in answering the problem was due to a lack of conceptual knowledge and strategic knowledge.

When all subjects already know each other and understand the answers of the exchange results or interaction of their thinking, then each subject wrote an answer on the answer sheet that has been provided. Answers already exist in the working memory and will be represented (rehearsal) from long-term memory to working memory to be manifested in a real form. It was found also subjects who are not confident to write the answer from the results of his own thoughts. This means that in the working memory of the subject the answer has been agreed together yet incomplete. This possibility occurs because not yet the recipient of the information still do not understand. Kieran, (2002) states that the submission of incomplete information if submitted by the informant will be considered back by the recipient of the information. This matter some of the subjects still looking back from the reply of his friend who has been stated in the real form. After reviewing the answers from his friends, then some of the subjects had written their own answers. Prusak, Hershkowitz, & Schwarz, (2012) argue that group work not only builds effective communication but also engages productive arguments in which students engage in various forms of mathematical representation and deductive reasoning. So the answers of all group members can develop from those already possessed with knowledge gained from the explanations of other students.

Writing answers from the agreement of all members of the group will lead to the way it represents the same. In fact, it turns out that each subject has a way of representing its own answer. The five subjects write it in their own way. Their knowledge and understanding of this answer have been encoded on their long-term memory. The students answer the questions given depending on the pattern of their interaction activities. The pattern of interaction activities has to do with the results of problem solving. This is in line with the results of the research investigated by (Barron, 2000, 2003; Lai & White, 2014; Watson & Chick, 2001)

CONCLUSION

The result of the students' interaction thinking in solving the open-ended problem begins with one group member giving statement and understanding of the problems provided. Then another group member responds and responds to the statement. The activities of sharing ideas, completing or giving feedback to other members' comments, asking questions that have not been understood, or giving this explanation occur when deciding on the answer to be agreed upon. As the discussion progresses, there is a thought interaction process to provide help from one subject to another. Subsequent activities decide the answer orally to be written by each subject. All subjects poured the agreed results on the answer sheet, but there were still doubts from some subjects on the answer, so the subject looked back at the answers of other subjects. Some subjects continue to write answers and in the end, each subject has a form of response in accordance with their respective understanding. The five subjects in this study to find answers by way of discussion orally represented in answer sheets, and have a variety of answers model.

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