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# REPRESENTATION TRANSLATION ANALYSIS OF JUNIOR HIGH SCHOOL STUDENTS IN SOLVING MATHEMATICS PROBLEMS

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***Abstract***

This research aims at analyzing the representation translation of Junior High School students in solving mathematical questions related to algebra. This research used descriptive qualitative approach. The focus of representation translation used in this research was the external representation which was a verbal translation into diagram, verbal into symbolic, symbolic into diagram, diagram into symbolic, and diagram into verbal. Based on the analysis of research findings, it shows that representation translation of students from verbal, symbolic, and diagram into verbal and diagram was not really mastered by the students. Meanwhile, the representation translation into symbolic was frequently used by the students although they were expected to do other translation rather than symbolic.

**Keywords:** translation, representation, mathematics problem

Mathematics representation is one of the general objectives of mathematics learning at school. It is implicitly stated in the Curriculum 2013 objectives in which a learning process is designed as a consideration between mathematics within and without numbers (picture, graphic, pattern, etc) (Kemdikbud, 2014). Picture, graphic, and pattern are other representation modes that can be used by teachers and students during the learning process. In line with that, NCTM in Principle and Standards for School Mathematics states that representation is the fifth standard of the process after problem-solving, reasoning, communication, and connection. Besides, the representation can be used as a means to strengthening, reasoning, and communicating an idea or a means to think cognitively (NCTM, 2000; Pantziara, Gagatsis, & Elia, 2009; Diezmann & English, 2001).

The definition of representation proposed by some educational experts (Goldin & Steingold, 2001; Kalathil & Sherin, 2000; Pape, 2001; Duval, 2006; Kaput, 1987; Kaput, 1989; Zhang, 1997) refers to an external mode to describe a mathematics idea constructed in one's mind. Representation refers to both the process and the result. Furthermore, it also means perceiving mathematics connection or concept in its mode and some other modes (NCTM, 2000). By representation, it is expected that the students are able to express the mathematics idea during the learning process. However, there are three things that students must require, those are: a) being able to create and use representation to organize, record, and communicate the idea; b) selecting, using, and translating various representation to solve problem; c) and applying representation to epitomize and interpret a phenomenon (NCTM, 2000).

Goldin & Kaput (1996) distinguish representation occurring into two steps, which are internal and external representation. Internal representation refers to a mental configuration that enables an individual, such as a student to comprehend or solve problems. On the other hand, external representation is the result to show everything that students do (word, graphic, picture, and equation).

Bruner (in Post, 1988) divide representation as enactive, iconic and symbolic. (1) Enactive representation is a sensory-motor representation constructed through action or movement. (2) Iconic representation is a representation applied in visual shadow, picture or diagram mode that describes a concrete activity or situation in enactive step. (3) Symbolic representation is associated with mathematical language and symbols. Lesh, Landau, and Hamilton (in Post, 1988) developed Bruner representation by adding "spoken symbol" and "real world situation". As a result, there are five types of representation, they are real scripts, manipulative models, static picture, spoken language, and written symbols (Janvier, 1987). From various representation existing, those models are often taught separately (NCTM, 2000), whereas students need to relate various representation modes based on the indicator of representation mastery in NCTM, in which the students are able to select, apply and translate miscellaneous representation modes in solving the problem.

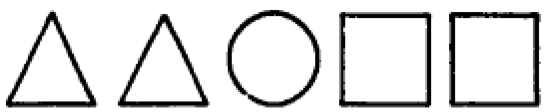
In addition, Janvier states that translation process is *"the psychological processes involved in going from one mode of representation to another, for example, from an equation to a graph"* (Janvier, 1987). It means the translation process is a changing process from one representation mode to another. Ainsworth (1999) defined mathematical translation as "*all cases when learner must see the relation between two representations*". On the other words, it means that the students are able to see the relation between the two representations. Translation is a cognitive process and relation from one representation mode to another different representation mode, without changing the notation object (Roth & Bowen, 2001; Duval, 2006; and Bossé, Adu-Gyamfi, & Chandler, 2014). Duval (1999) divides the cognitive process in translation into two categories. Those categories are processing and conversing. Processing is meant as a translation between the same representation mode or type, for instance changing one algebra mode into another algebra mode that has the same value. Meanwhile, conversing is a translation between two or more than two different representation modes, transforming a linear equation into a graphic. Translation term used in this research refers to the converting process, which is transforming between different representations in solving the mathematical question.

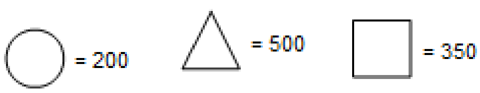
There are some researches focusing on translation process of students and identifying mechanisms through which cognitions and relations between representations that had been carried out before (Bossé et al., 2014; Duru & Koklu, 2011; Hwang & Chen, 2007; Clement, 1982; Duval, 2006; Kaput, 1987; Knuth, 2000; MacGregor & Stacey, 1993; Sims-Knight & Kaput, 1983). For example, Bossé et al., (2014) conducted a research focusing on translation activity of students from graphic to symbolic based on the level of mathematics ability of students. Besides, Duru & Koklu (2011) also carried out a research about translation between text representation mode and algebra. In this research, it is revealed that students found difficulty in translating from text mode into an algebra equation using the symbol. Students also found it difficult to translate from symbolic mode into the text because of the weakness in reading comprehension. Hwang & Chen, (2007) state most students do not really understand the importance of representation modes connection in solving a mathematical problem. The students apparently only apply one representation mode, such as procedure taught by the teacher. When encountering difficulties in solving the problem, the students rarely try to use different representation in solving them.

Some translation researches above only focus on one representation mode. Moreover, those researches were not deeply discussed in each mode of representation. Therefore, the objectives of this research are to analyze the translation of student's representation further in solving mathematical questions by using the various translation of representation modes. The representation translation applied in this research used 3 modes, they are verbal, symbolic and diagram.

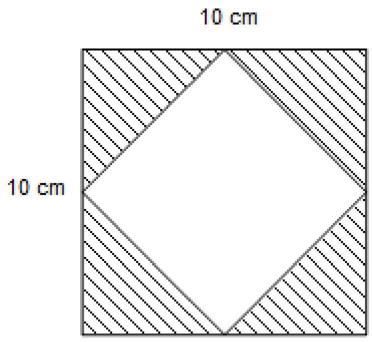
## METHOD

This research used descriptive qualitative approach. This research was aimed to describe the representation translation process of Junior High School students in solving the mathematical question. There were 6 items of mathematics question given. The types of these items were verbal, diagram, and symbolic (algebra). The focus of this representation translation used in this research was the external representation which was a translation of verbal to the diagram, verbal to symbolic, symbolic to verbal, symbolic to the diagram, diagram to symbolic, and diagram to verbal. There is coding in this translation among these representations in this article, V as verbal, D as a diagram, and S as symbolic. If there is any translation from verbal to symbolic, then the code will be V→S, as well as the other representation translations. The test instrument showed below.

1. A frog is stuck in an empty barrel. The barrel's height is 2 m. The frog is trying to get out by jumping over the barrel wall. Every minute the frog jumps 30cm long, but instantly after jumping he always slips down 5cm long. How long does it take the frog to get out of the barrel? ( *answer in the form of a diagram* )
2. Andi and Ira pick apples and strawberries in agro-garden. Andi picked 3 kg of apple and 2 kg strawberry. While Ira picked 1.5 kg of apple and 3 kg strawberry. If Andi paid Rp110.000 for the fruit picked and Ira paid Rp105.000, - then what is the price of each fruit per kg? ( *answer in symbolic form* )
3. $5a = 2a + 27$. Find the value of $a$? ( *answer in verbal form* )
4. $2m = 10m - 24$. Find $m$? ( *answer in the form of a diagram* )
5. Determine the sum of all the following image 

   If 

   ( *answer in symbolic form* )
6. Measure the width of the shaded area in the picture below! ( *Answer in verbal form* )




The method used in this study was a task-based interview. It was conducted by giving mathematics questions as it is mentioned in the appendix. The subjects of this research were two students of Junior High School in Malang that have perceived material about two variables linear equation. Those subjects live near the researcher. Thus, the researcher chose them as the subjects for accessibility reason. The researcher took a role as the main instrument assisted by a supportive instrument which was a test, and completed with interview.

## RESULT AND DISCUSSION

The following explanation is the results obtained by the two subjects; they were S1 and S2.

***Students' translation from V→D***

In question number1, the mode of the question was verbal and the students were asked to present the solving in the diagram. In solving question number 1, S1 and S2 did not translate the question into answer directly to the diagram mode. Both subjects did the questions in symbolic mode first and then made the diagram. The diagram in Figure 1 made by S1 and S2 were almost similar.


Figure 1. S1's Answers of Question Number 1

S1 could do question number 1 correctly by using standard algorithm first. Based on the interview, S1 subtracted 30 with 5 and the result was 25 cm. Then, she converted 2 m into 200 cm. Next, S1 divided 200 cm with 25 cm and the result was 8. Then, S1 made a diagram as it was required in the question. S1 made the diagram based on the result that she obtained, not the actual answer of the question. Thus, she drew 8 circles that were linearly connected, where the value of each circle was 25.

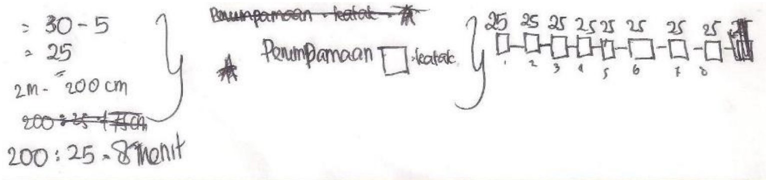



Figure 2. S2's Answers of Question Number 2

Similar with the working result of S1, S2 solved question number 1 with standard algorithm first. Then, S2 made a diagram based on the algorithm calculation done before. The diagrams made by S1 and S2 were based on calculation, not the verbal question. S2 made the diagram by analogizing square with frog, while S1 made 8 squares which were linearly connected.

***Students' translation from V→S***

In question number 2, the question was in verbal and the students were required to present the solving in symbolic mode. In solving question number 2, S1 and S2 analogized each item with a different variable. S1 and S2 performed different methods to solve this equation.

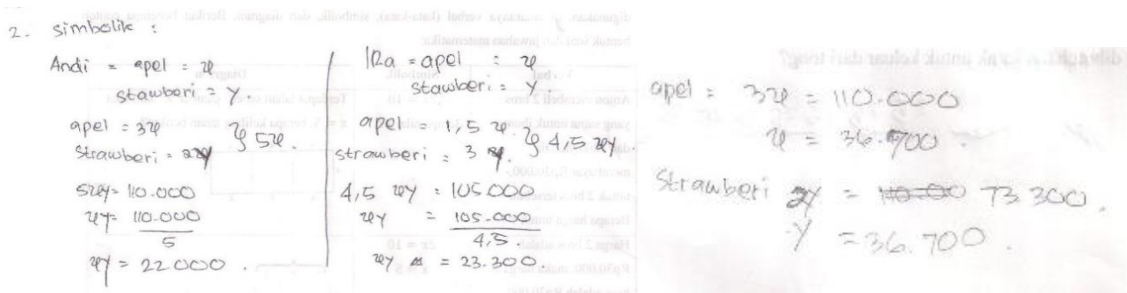



Figure 3. S1's Answers of Question Number 2

| | | |
|---|---|---|
| *P* | : | *How do you understand question number 2?* |
| *S1* | : | *For the apple, Andi pluck apple 3 kg and 2kg strawberries and Andi pay it all 110 thousand, so that means each fruit for 1 kilo is 36.700* |
| *P* | : | *Why?* |
| *S1* | : | *Because ... ee.... that... the apple for 3 kilos, then the apple let a $x$* |
| *P* | : | *Why do you think is $x$?* |
| *S1* | : | *Because when I studied in school, I usually use $x$* |
| *P* | : | *Can it use letter $a, b, c$ ?* |
| *S1* | : | *Yes, it can, but I choose $x$* |
| *P* | : | *The strawberry use $y$?* |
| *S1* | : | *yes* |
| *P* | : | *Why is not $x$ too?* |
| *S1* | : | *The fruits are the difference so I make the letter different* |
| *P* | : | *Owh. Then why would 5xy = 110.000?* |
| *S1* | : | *Because 3x plus 2x it's the same as 5x ... 5xy* |
| *P* | : | *5xy? 3x plus 2y so 5xy huh?* |
| *S1* | : | *yes* |

S1 performed some inappropriate translations in solving questions number 2. In translating the fruit in the question (verbal) into a variable (symbolic), S2 made the variable as the object (fruit) not the price of the fruit. S1 makes variable permissions based on what is taught in schools. Besides, from the interview and the result of S1, it is found that S1 translated from function 3x + 2y to 5xy. S1 also made a mistake in algebra calculation accordingly she did not realize that she obtained the wrong answer.

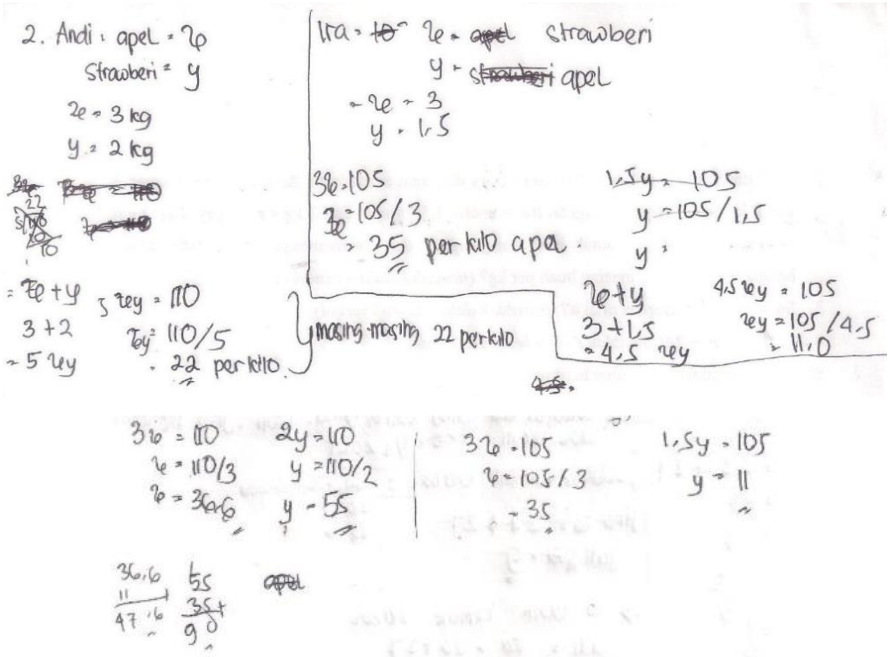



Figure 4. S2's Answers of Question Number 2

| | | |
|---|---|---|
| *P* | : | *Ok, next, the number 2, how you understand the problem?* |
| *S2* | : | *Andi .. mm .. I think the apple is x and strawberry is y* |
| *P* | : | *This you mean x, it means x is apple or apple price?* |
| *S2* | : | *Apple* |
| *P* | : | *Not the apple price?* |
| *S2* | : | *Nope* |
| *P* | : | *Then how come 3x = 110 and 2y = 110 too, why it is similar?* |
| *S2* | : | *Yeah, that's 2 from what it's called strawberry, so 2y = 110, so y it's 110/2 so 55* |
| *P* | : | *So each one is equated 110?* |
| *S2* | : | *Yes* |

Almost similar with the answer of S1, S2 also made some incorrect translations in solving question number 2. S2 translated the fruit (verbal) into a variable (symbolic), not the price of the fruit. S1 was inconsistent in translating. On her worksheet, she translated variable as the weight of the fruit (x= 3kg, and y=2kg). However, during the interview, S1 stated that the fruit is the variable. S1 also made a mistake in algebra calculation so she did not realize that she made the wrong answer.

***Students' translation S→V***

In question number 3, the mode was symbolic and the students were required to perform the solving in verbal mode. In performing question number 3, S1 and S2 did not translate the question directly into the written verbal mode. Yet, through oral verbal mode, S1 and S2 could explain their work. Both subjects performed the question into the symbolic mode and provided a conclusion in written verbal mode.

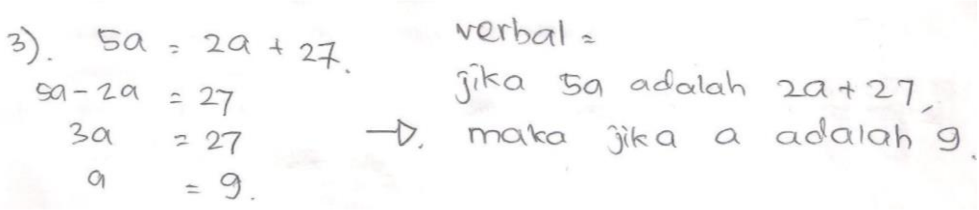



Figure 5. S1's Answers of Question Number 3

*P* : *Ok, then go to number 3, mmm .. number 3 is this how do you? From here why, from the first line to the second line why is it so like that?*

*S1* : *Because of the variable to the number is different, if the variable on the left then the number is on the right*

*P* : *Then why is 2a so min, negative?*

*S1* : *Since it is number to a variable, so if the number is added, means that the variable is negative.*

*P* : *So negative, What is the meaning of the variable number?*

*S1* : *Accordingly, the displacement would be negative if it is positive and positive if it is negative*

*P* : *Then this is the 3rd and 4th row*

*S1* : *The 4th is equal to the 2nd since the 2nd is lesser and the 3rd is the result. Then, the 4th is the 3rd, 3a=27, so the 4th is a=9 since 3a and 27 both are divided by 3*

*P* : *Why is it equally divided by 3?*

*S1* : *Because the variable should be 1*

In her work, S1 solved question number 5 by doing algebra calculation (symbolic). S1 could do the algebra calculation linear equation correctly. Yet, S1 did not completely comprehend the concept of linear equation algebra. As for the example, based on the interview, S1 mentioned the segment shift to collect the mode of the variable. S1 did not do a translation from the question (symbolic) into written verbal yet S1 could explain it verbally. S1 translated into verbal mode based on the calculation (symbolic) that she performed first to obtain the answer.

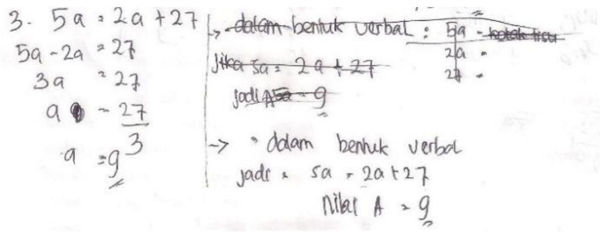



Figure 6. S2's Answers of Question Number 3

*P* : *As for question number 3, how do you explain the first until the second row?*

S2 : *The equation is 5a=2a+27, I moved all the number with variable a to the left segment and the one which has no variable in the right. It means that 5a-2a=27, thus 3a=27. To obtain a, 27/3 and it is 9.*

P : *Why did you move the number with variable?*

S2 : *So, if it is negative, it should be moved in the left, and if it is positive, it should be moved here and become negative.*

P : *Why did you move with the different sign?*

S2 : *Because the segment is moved so it is changed*

P : *What is your reason change the sign if it is moved?*

S2 : *I don't know, if it is moved to a different segment, it becomes negative, although it is positive and vice versa.*

P : *And, here, why did you divide it by 3? Not reduced?*

S2 : *No, to obtain the result, so 27/3*

S2 solved question number 3 by calculating symbolically. S2 did not translate it into written verbal. However, during the interview, S2 could explain verbally. S2 could correctly do the procedure of algebra calculation, yet S1 did not completely understand the concept of linear equation algebra. S2 collected the variable modes into one segment equation without knowing the reason.

***Students' translation S→D***

In question number 4, the question was in symbolic mode and the students were asked to solve this question in diagram mode. In solving question number 4, S1 and S2 did not also directly translate the question into a diagram. Both subjects solved the question in symbolic mode and provided a conclusion in diagram mode.

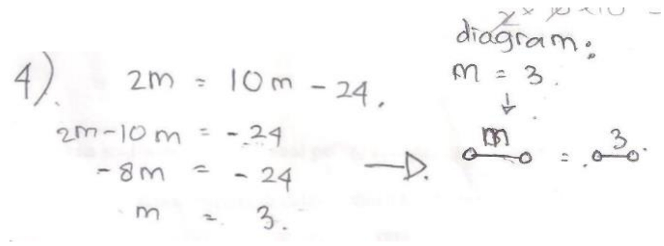



Figure 7. S1's Answers of Question Number 4

P : How do you understand the question number 4?

S1 : Similar to number 3, I moved the number with a variable in the left. So 2m-10m and the right is -24. So, I obtained -8m=-24, m=3

P : How about the diagram?

S1 : I assumed as what I write and it is equal to 3

P : Why did?

S1 : It was what I thought

P : So, you calculated first then drew the diagram?

S1 : Yes

P : Why did you calculate first than draw the diagram?

S1 : I confused when I drew the diagram first, and to make it quick, I calculated first

*P* : How if we use another form? Could you?

*P* : Hmmmmm... I think I can, but it was what I thought.

*S1* : Why did you draw two bullets and line?

S1 solved the question number 4 by doing algebra calculation first, similar with question number 3. Then, S1 drew a diagram based on the calculation result. The answer was correct, yet S1 failed in explaining the meaning of the diagram she drew.

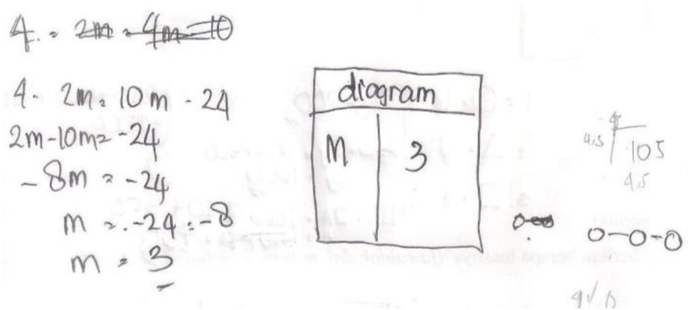



Figure 8. S2's Answers of Question Number 4

*P* : *(pause) ok, next the number 4, number 4 is how you do it how can that be?*

*S2* : *Yes that have the same m variable like number 3 was moved to the left side*

*S2* : *Then it's positive, move to the left side and becomes negative, so 2 m minus 10m so negative 8, then this is also negative, then m = 24 / -8 so the result is 3*

*P* : *Then what is the diagram means?*

*S2* : *I formed like this one then it'll be easier and quicker, so m = 3*

*P* : *So there's just the letter then the result?*

*S2* : *Yes*

*P* : *How if it is another form?*

*S2* : *It Could be anyway*

*P* : *How*

*S2* : *For example, like this (draw) same result later (pointing)*

*P* : *So What is this 3?*

*S2* : *It is 3, the result of m*

*P* : *Why did you draw two bullets and what do you mean by 3?*

*S2* : *It's 1 hmmm .. it is m*

Almost similar with S1, S2 solved the question number 4 by doing algebra calculation first. S2 could solve the question number 4 correctly through algebra calculation, yet, she did not comprehend the concept. After that, S2 drew a diagram from the result she obtained. Nevertheless, S2 failed in explaining the meaning of the diagram she made. S2 also provided with the other diagram for the solving of that question. Nonetheless, at the end, S2 was confused with the diagram she made.

***Students' translation D→S***

In question number 5, the mode of the question was in the diagram and the students were asked to provide the solving in symbolic mode. In solving question number 5, S1 and S2 performed the solving differently. S1 and S2 also interpreted the meaning of the question differently.

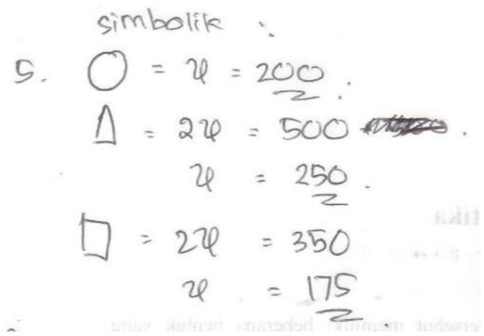


Figure 9. S1's Answers of Question Number 5

| | | |
|---|---|---|
| *P* | : | *Then, why did you write x all? Not differentiate it?* |
| *S1* | : | *Because all the shape are similar, to make it easy* |
| *P* | : | *How about a different symbol?* |
| *S1* | : | *Yes, we can* |
| *P* | : | *(pause) so, if the symbol is similar or different, both are correct?* |
| *S1* | : | *,Mmm so...the questions are not similar, so it can be different. For example, if the question is summed, so it must be similar. Later, if the variable is different it can be solved quickly* |

S1 solved the question number 5 by translating into symbolic in each picture (plane figure) given in variable x. On the worksheet of S1, S1 interpreted the question correctly, thus she answered the question based on each pictures, not all pictures in the question. Therefore, S1 provided the same variable equation for each picture.

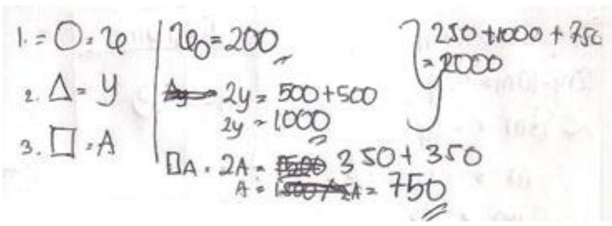


Figure 10. S2's Answers of Question Number 5

| | | |
|---|---|---|
| *P* | : | *That's why I put different letters or variables? Why is not the same?* |
| *S2* | : | *No, it is not* |
| *P* | : | *Why?* |
| *S2* | : | *To differentiate, if it is similar it is confusing* |
| *P* | : | *Is it because the shape is different?* |
| *S2* | : | *Yes the shape is different* |
| *P* | : | *If the shape is similar, can it be also similar?* |

*S2 : Yes, it can*

*P : So what is the value of this figure?*

*S2 : 2000*

*P : Why is it so?*

*S2 : Because I added the x 250 y 1000 that a 750 so it is 2000*

S2 translated question number 5 into the symbolic mode by analogizing it into the different variable mode. S2 used x for circle, y for a triangle, and A for the square. Then, S2 did algebra calculation as much as variable known so she obtained the answer. S2 added all values of the variable to obtain the answer.

***Students' translation D→V***

In question number 6, the mode of the question was in the diagram and the students were asked to provide the solving in variable mode. In solving question number 6, S1 and S2 solved it correctly regardless the different methods.

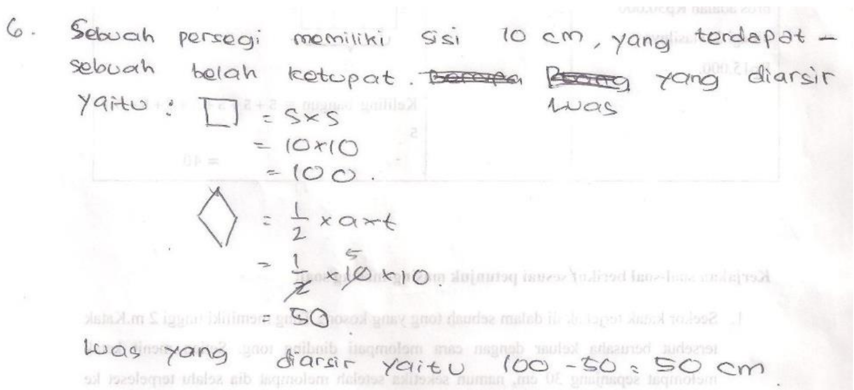



Figure 11. S1's Answers of Question Number 6

*P : Why is it reduced?*

*S1 : Because it's right there in the square there is a rhombus, then it's not a rhombus not all eee... covering the square, there is part of the blank, the empty part is the difference*

*P : So, based on your answer, between using verbal to symbolic and diagram, what method is easier?*

*S1 : It depends on*

*P : If from here? Which one is easier than usual*

*S1 : Use the symbolic one*

*P : Use the symbolic, why?*

*S1 : It's esier and I already understand the variables*

S1 could answer question number 6 correctly. However, S1 answered the question by translating symbolically first, even though it is expected to solve it in verbal mode. The verbal mode occurred in S1 translation was not in written mode, yet verbally. S1 solved the question by finding the width of the square and rhombus first. After that S1 subtracted the width of the square with the rhombus width hence she obtained the result.

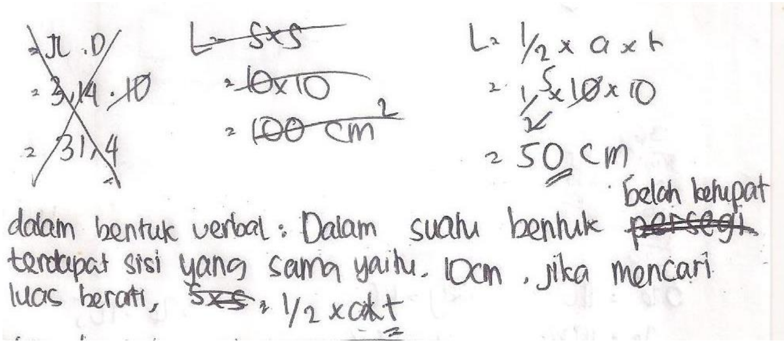



Figure 12. S2’s Answers of Question Number 6

| | | |
|---|---|---|
| *P* | : | *how did you obtain the answer* |
| *S2* | : | *This is the width of the half multiplied by the height multiplied, half multiplied by 10 times 10 so 50,* |
| *P* | : | *Why did you use that way?* |
| *S2* | : | *Well ... so ... it's because it's a rhomb* |
| *P* | : | *What are you looking for? The shaded? not the rhombus? How can you use rhombus?* |
| *S2* | : | *Yes because it looks the same like a rhombus* |
| *P* | : | *How can it be the same* |
| *S2* | : | *Samilar if it is inserted (pointed shaded)* |
| *P* | : | *So, this shaded is inserted to the rhomb?* |
| *S2* | : | *Yes* |
| *P* | : | *It looks like a rhombus* |
| *S2* | : | *Yes* |
| *P* | : | *How can you know that it is similar?* |
| *S2* | : | *Yes, when it’s folded, it’ll be the same* |
| *P* | : | *Oh folded so, like origami so?* |
| *S2* | : | *Yes* |

In solving the question number 6, S2 used a different method from S1. S2 solved the question by translating inthe to verbal mode, in written, verbal and symbolic. S2 found the shaded width by discovering the width of the rhombus based on the known square side. S2 assumed that the shaded figure was same as the rhombus in the picture if it was folded.

*Discussion*

The translation of representational forms can help the students to more easily solve the problem. This is in line with Eraslan's (2008) research in helping students to define goal representation, reducing the level of abstraction of a representation. A translation mode from a symbolic or diagram to the verbal mode is not really known by the students in the learning process. Therefore, the students do not really master in doing the translation into verbal mode as well as translation symbolic or verbal into the diagram. The students do not completely understand the meaning of the diagrams they made. Thus, teachers need to introduce more about representation modes to the students. This opinion is in line with McCoy (in Elliot, 1996),"... if the student is given a function such as y = 4x – 2, they should be able to describe question situations for which the equation would be used". It is also supported by Rittle-Johnson, Siegler, & Alibali (2001) statements that conceptual understanding and problem representation are mutually underlying. With a broad knowledge of various representation modes, students are able to interpret various questions, in verbal, symbolic or diagram and do translation among representation modes.

On the contrast, a translation made from verbal or diagram into symbolic is often found by the students. Yet, the students lack in mastering linear algebra concepts accordingly there are some mistakes in solving questions into that mode, especially in two variables linear equation. The research data shows that the students actually understood the question by making a function in variable modes. Nevertheless, the students should be reminded and given a comprehension. It is in line with Kriegler (2007) who state that In short, being fluent in the language of algebra requires understanding the meaning of its vocabulary (i.e. symbols and variables) and flexibility to use its grammar rules (i.e. mathematical properties and conventions). Thereby, the students' mastery in using various representation modes and translation among representations should be the concern of teachers to introduce them to the students.

## CONCLUSION

Based on the problem issued in this research, findings, and discussion, it can be concluded that: First, students do not really master representation translation from verbal, symbolic, and diagram mode into verbal and diagram mode. Students do not understand the meaning of the diagram that they made. Students also difficult in translating written to verbal mode. However, students are more able to to translate into the verbal mode. Second, representation translation into symbolic is more often used by students even though they are asked to do the symbolic translation. Nevertheless, the students' conceptual understanding in symbolic translation is still poor and overall they do not realize that they made some mistakes.

However, the test questions used in this research is not really reliable since there were some students who interpreted the questions ambiguously. Thus, it is expected for the future researchers to improve the quality of the test questions to reduce the misinterpretation of the questions. In addition, it is also expected for the teachers to give a deeper understanding of the use of various representations. Besides, teachers also need to emphasize translation understanding about some various representations so that there will be objects consistency in doing translation toward each representation modes. To know the students' ability deeper, it is better to have a discussion with the teachers, compared to do a brief observation and research by ourselves.

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