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PRE-SERVICE MATHEMATHICS TEACHERS' PREFERENCES IN USING MULTIPLE REPRESENTATION OF WORD PROBLEM SOLVING

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Abstract

Multiple representations have an essential role in the mathematics learning process for the students, pre-service and in-service teachers. It is important for them to understand the concept of mathematics and mathematical problem solving particularly in word problem-solving. However, pre-service teacher and in-service do not fully notice of using a variety of representations in the learning process. Thus, the purposes of this study are intended (1) to find out the type of multiple representations used by pre-service teachers to solve the word problem by their own, and (2) to figure out how they teach the process of word problem solving to the students. The participants of this research were the sixth-semester students of the Mathematics Education Program. Their written responses were analyzed and interviewed (semi-structured interview). The result showed that the types of multiple representations used by pre-service teachers are symbolic and verbal representations. Different multiple representations are used to help the student in word problem solving, even though pre-service teachers did not fully aware to apply them in their teaching activity. Hence, sharing the knowledge and the usage of multiple representations for pre-service mathematics teachers are urgently required in the real classroom setting.

Keywords: Preferences, Multiple Representation, Pre-service Mathematics Teachers, Word Problem

Multiple representations have an important role in mathematics learning. Thus, it becomes substantial topics in learning and research of mathematics education in the recent years (Cai, 2009; Dreher, Kuntze, & Lerman, 2016; Durkaya et al., 2011; Lin, Wilson, & Cheng, 2013). Multiple representations become very significant or important because it enables students to comprehend the mathematical concept deeply (Duval, 2006; Goldin & Shteingold, 2001; NCTM, 2000; Tripathi, 2008). Hence, the teachers should pay attention to use multiple representations when they teach this particular topic.

The capability of using the multiple representations in teaching and learning mathematics can also help the students to solve mathematics problem (Bal, 2014; Hwang, Chen, Dung, & Yang, 2007; NCTM, 2000; Stylianou, 2010), such as a word problem. However, the word problem is sometimes difficult for students to interpret it (Gagatsis & Elia, 2004; Gagatsis & Shiakalli, 2004; NCTM, 2000). Therefore, the teachers should have the ability to represent the word problem's meaning into different representation models so that the students can understand the problem's idea easily. By doing so, the students will be able to solve the problem correctly.

Looking at the importance of multiple representations in conceptual understanding and problem-solving in mathematics, the teachers should be fully aware of using some teaching strategies which fit multiple representation learning processes. That is because, with the various representations, it will be possible to accommodate the different learning preference of the students (Choike, 2000). However, teachers tend to use single representation and less conscious of the importance of multiple

representations in their learning (Stylianou, 2010; Zhe, 2012). This is a need to be concerned by education practitioners to raise mathematics learning, as what Kilpatrick, Swafford, & Findell (2001) stated that by using multiple representations, the mathematical ideas can be further improved in mathematics learning.

Multiple representations are generally defined as providing the same information in more than one form of external mathematical representation (Goldin & Shteingold, 2001). Hence, understanding other concepts of mathematical problem solving can be done in various ways. There are several models of representations depending on the theory that is used. Lesh, Post, & Behr (1987) identified five distinct modes of representations that occur in mathematics learning and problem-solving; they are (1) real-world situations, (2) manipulative model; (3) pictures, similar to manipulative model that internalized as a picture; (4) spoken symbols, which can be everyday language; (5) written symbols, in which specialized sentences and phrases take place.

Another study shows that mathematical representation methods in problem-solving are also divided into five different scopes (Matteson, 2006), i.e. (1) numerical representation, focuses on certain numerical values in various formats; (2) graphical representation, which contains six distinct visual representations – picture graph, model, horizontal graph, vertical graph and coordinate graph; (3) verbal representation, it involves written language to comprehend, describe, analyse, examine or contemplate numerical representation, algebra or graphic that does not enclose a short phrase about the instruction to solve the problem; (4) symbolic representation, it focuses on symbolic notation and covers on variables and formulas application. Five symbolic representations that can be found are equations, equation expression, algebra, algebraic expression, and formula; (5) dual representation, it contains two categories of representation that have been stated before.

The importance of the awareness of using multiple representations does not only require the teacher's concern, but pre-service teachers also need to pay attention to using multiple representations in mathematics learning as well, either by giving an understanding of the mathematical concepts to the students or by solving a mathematical problem. It is because the quality of pre-service teachers depends on their understanding about the content knowledge and pedagogical content knowledge in mathematics learning. According to Shulman (1986) that Mathematical Content Knowledge and Pedagogical Content Knowledge are part of effective mathematics instruction. Gulkilik & Arikan (2012) assert that the pre-service teacher is an in-service teacher in the future so that the importance of understanding multiple representations in teaching and learning process should be considered.

There is much research concerning the representation skill of pre-service teachers in problem-solving (Bal, 2014; Durkaya et al., 2011; Son & Lee, 2016). The result of the previous research, Son & Lee (2016) proposed that in word problem-solving, pre-service teachers used symbolic representation, graphical representation, and written description. Then, as the word problem is taught to their students, pre-service teachers used various graphical representations. According to the result of Bal (2014) stated that pre-service teachers are capable of applying lots of representational types in solving the

mathematical problem, however, they are frequently found to use verbal language and algebraic representation. Durkaya et al (2011) on his research stated that pre-service teachers are found to have troubles in choosing the determiner, so they use a numerical and algebraic representation which are actually types of representations.

Based on the previous research, the pre-service teachers use some different representations in mathematical problem-solving. Then, when it is taught to their students, they use some graphical representations. Therefore, it is required to conduct a study to enhance the knowledge about the representation that might appear to pre-service teachers trying to word problem-solving for their own. Whether the result is in line with the previous research or not. Furthermore, it is needed to identify the types of representation that might occur in problem-solving before it is taught to the students and how pre-service teachers present it during the learning practices in the classroom. The further research is needed to establish the basic and the idea in developing the quality of mathematics education. Besides, this research is necessary to be conducted just as Stylianou (2010) stated that there is still less evidence required to support the professional development program in preparing the teachers and pre-service teacher in relation to representation.

Based on the discussion presented above, the objectives of this research are: (1) to find out the types of multiple representations used by pre-service teachers to solve the word problem by their own; and (2) to figure out how they teach the process of word problem solving to the students.

METHODS

Participants

The participants of this research consisted of 40 sixth semester students in Mathematics Education program academic year of 2014/2015 in one of the Private University in East Java. Three pre-service mathematics teachers were chosen after giving tests to be interviewed with considering their written response.

Instrument and Data Collection

The research instruments are modified from the research of Son & Lee (2016). The first question is what representation type pre-service mathematics teachers use to solve the problem. The second question is about how pre-service teachers use multiple representations in word problem solving that will be explained to the students. The research instruments are presented in Figure 1.

The students in a seventh grade are planning to watch the Basketball match. However, after being confirmed, only two-thirds of the students are allowed to watch the match. From, the number of students that are allowed to watch, half of them are going by car.

1. What part of all of the seventh-grade students who are going to watch the match by car?

2. Imagine that you are teaching the seventh-grade students in Junior High School, think about the given question and how will you answer it so that your students can easily understand and are able to solve it? If it is so, for example, each student has a different learning style, how can you explain the process of problem-solving towards your students?

Figure 1. Research Instrument

Based on Figure 1, the second question is about how the pre-service teachers teach the process of word problem solving for students' different learning style, intentionally, to encourage pre-service mathematics teachers to use multiple representations in their learning process. This is under consideration that students later can use vary representation in word problem-solving.

This research instrument is given to the research subject consisting of forty pre-service teachers in the classroom in thirty minutes as the time limit. After analyzing their written responses, a semi-structured interview for three of pre-service teachers is conducted. The interviewed topic is related to the given questions. Some examples of the semi-structured interview in this research instruments are:

- Why do you use the [particular] representation when you will teach your students?
- What are in your mind when you solve the problem in such a way?
- Is there any other strategy that will be given to the students who have different learning style?

After interviewing, pre-service teachers were given tasks to answer identical questions that have been given before. The task was focused on the second question in order to know how they use multiple representations for solving the word problem.

Data Analysis

Data analysis of this research is based on two parts, the first part is analyzing the written response from mathematics pre-service teachers consisting of forty respondents. Then, the second part is analyzing the interview result from three participants of pre-service teachers related to their written response about the process of word problem solving by their own. Further, the interview is done in order to find out the reason why pre-service teachers use particular representations in their written responses. The next part is pre-service teachers are interviewed about their strategies in using multiple representations in word problem solving to help students understand about the questions when it is related to students learning the style.

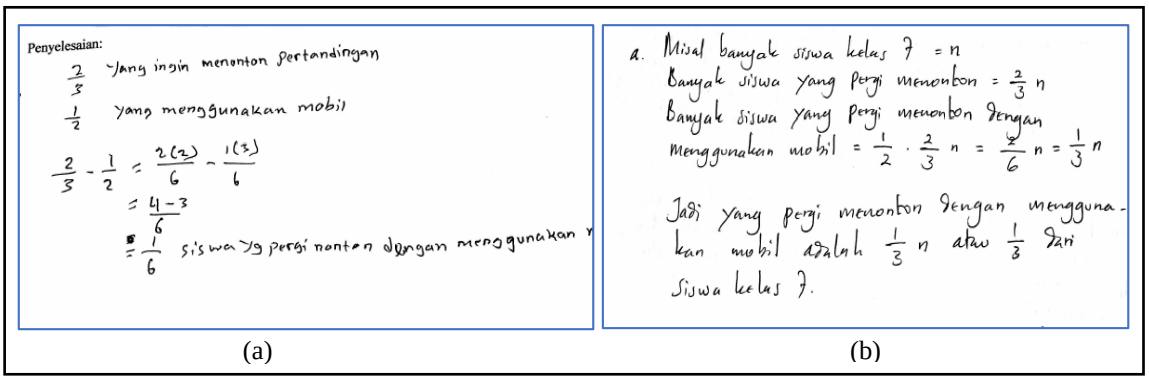
After to the three pre-service teachers are interviewed, they are given task with the same questions which are given before. This is done in order to get a deeper understanding about pre-service teachers' strategies in using multiple representations in word problem solving when the answer is well considered and also find out that pre-service teachers use another representation in solve the word problem or not. The reason why pre-service teachers are given the task is that preparation is needed in learning design before the lesson is started.

The use of representation in this research is taken from other related literature (Bal, 2014; Lesh et al., 1987; Matteson, 2006): (1) verbal representation, pre-service teachers are expected to understand the problem related to mathematics by expressing a solution or a result from the problem verbally; (2) graphical representation, that is, using shape, diagram, line number to explain the problem; (3) symbolic representation: focuses on symbolic notation and covers the use of varied and formula. There are five symbolic representations that are found, which are an equation, expression, algebra expression, and formula; (4) numerical representation: using tables or matrix to explain the problem; (5) real-world situation, where the knowledge is set around the situation in the real world.

RESULTS AND DISCUSSION

The use of Multiple Representation in word problem solving for own pre-service teachers.

Based on the written response from the participants related to word problem solving of question number one, pre-service mathematics teachers applied symbolic and verbal representation. Here is the example of pre-service teacher response on item 1, as shown below:



(a) (b)

Figure 2. Participant's response in problem 1

Figure 2. shows that pre-service teacher used symbolic and verbal representation, but in Figure (2a), there was a mistake operation in solving problem, thus the answer was incorrect. Then, based on the written response of pre-service teachers, there was not found other representation such as graphical representation used (length model, area model, and set model) in solving the problem.

This result differs from the finding of Son & Lee (2016) which pre-service teachers use symbolic representation, graphic representation, and written description in word problem-solving. The difference in the result of this research with the previous research is probably caused by their assumption toward word problem-solving question number one is resolved for themselves, therefore they prefer using symbolic representation, though, some students use verbal representation. So, it can be concluded that in representing word problem solving, pre-service teachers represent it in the form of symbolic and verbal representation. From this research, it is found that pre-service teachers solve the problem from a word problem to symbolic and verbal representation.

Next, when we take a look about the operation done by the pre-service teachers in representing world problem, the way they use the operations are also different in the translation of the operations themselves. From the interview, it is found that pre-service teachers have different meaning about the questions. So that they have differentiated representation to solve the word problem. The reason why they have different meaning is from the statement in question such as the meaning of, "Half of the two-thirds". Each pre-service teachers represent "of" from the question in a different form. There are 35% of pre-service teachers assumed that in solving $\frac{1}{2}$ of $\frac{2}{3}$ By using division operation, so that they divided it by $\frac{2}{3} : \frac{1}{2}$. The other 10% used addition. They simply added $\frac{2}{3}$ and $\frac{1}{2}$, while the other 35% used subtraction. The rest (20%) pre-service teachers used multiplication. This situation caused by some pre-service teacher failure to interpret word problems into other representation forms. This is

consistent with Son & Lee (2016) that some pre-service teachers are failing in interpreting the word problem, and also make mistakes in the formulation or in computation multiplication fraction.

The variation response occurred in the study need to be seen as an important thing. So, it is for teachers to understand about mathematics concept correctly, because when the interpretation of word problem is wrong then the representation will be wrong. Therefore, the teacher's competence in translating the representation relates to their success in solving a mathematics problem. This is in line with Villegas, Castro, & Gutiérrez (2009) that there is a significant and positive relationship between the success of problem-solving and their ability to use the representation. So, based on result above, the representation type used by pre-service teachers in word problem-solving in their own is a symbolic and verbal representation.

The use of Multiple representations of word problem-solving in mathematics learning.

Regarding how pre-service mathematics teacher teaches students to solve the second problem, it is found that pre-service teachers use some representations form, they are symbolic, verbal and graphical representations. The first participant solves the word problem by using graphical representation (draw a circle that illustrates a bread) with verbal and symbolic representation.

The response is illustrated in the figure below:

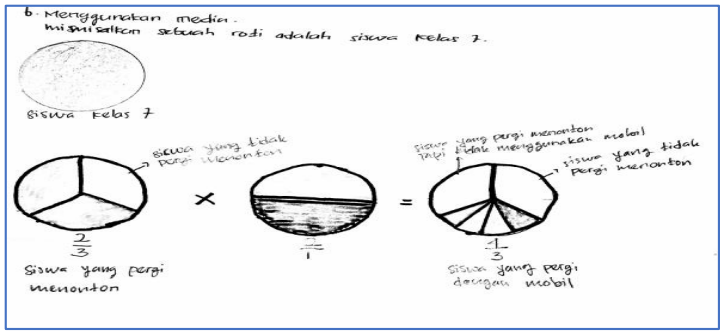


Figure 3. multiple representation produced by Participant 1

Surprisingly, even though the pre-service teacher is able to use different multiple representations, due to her lack of Mathematical Content Knowledge. She produces a wrong computational answer. The reason why she uses the representation, pre-service teachers said that it is because Junior High School students still need an example which is closely related to their world or real life. The following are illustrative of the first participant attempts to use graphical, with verbal and symbolic representation in word problem-solving.

Researcher: Why did you use a picture to illustrate bread?

Participant 1: I think seventh-grade students still have an elementary school mind so that with that concrete, it will be easier to understand.

Researcher: Is there any other methods?:

Participant 1: I used the same way as I solve the previous one (symbolic representation)

Based on the pre-service teachers' response, it is found that the pre-service teacher use symbolic and graphical representation (under verbal and symbolic representations) for solving the problem number two, but the answer given is still based on the way she solves for their own and she still confuse how to teach the students, moreover how to teach word problem solving to different students' learning style. The second participant responded that she used symbolic and verbal representation to solve question number two. In fact, she did not think how to teach the way of solving question number two for students. The second participant said that" *if I solve the problem, I will use that (symbolic and verbal representation). However, if I have to teach it to the students I have to read it again, have to learn how to teach it coherently with another method*." Then, the second participant also was given a task. Dealing with students' learning style, the pre-service teachers tend to use different representations, i.e she would use a rectangle to help visual students, take and advantage of mathematical aids for kinesthetic students, and explain the material through question and answer for audio learning students. However, the use of rectangle and mathematical aids were not detailly explained in solving the word problem. She uses different ways at glance.

Similarly, the third participant also used symbolic and verbal representation to answer the second question as the second participant did. The different existed when she was asked whether she used other strategies to solve the item. Fortunately, illustration or colourfull picture to help visual learner. She also adapted teaching strategies such as simulation and use learning media for kinesthetic learners. She preferred to engage the student in group class discussion to assist the audio learner in solving the problem. The observation showed also that she persistently used media for the learning process, but the learning media are not explained in detail.

Based on the interview, pre-service teachers realize that they will make use of multiple representations to help students in word problem-solving. But, they are still not optimal in using representation forms to word problem-solving. This is in line with Durkaya et al (2011) who stated that not all pre-service teachers realize the relation among representations.

Some pre-service teachers said that they did not think about how to use a different representation, but in fact, they are able to use symbols, verbal, graphic representation and learning media. However, they do not know which learning media is relevant to solve the problem yet. So it can be said that pre-service teachers already know that teaching 7th-grade students can be better if they supported by using learning media or manipulative model, but they did not specifically state which learning media can be used to solve the questions for students to understand how to comprehend the item solved.

Based on the data analysis, pre-service teachers had enough understanding to solve the word problem but they got difficulty when they came to share this knowledge with the students. They still do not think that latter they will be teachers who need to have the pedagogical content knowledge, and ability to use multiple representations in their learning. Hence, pre-service teachers solve the problem as an individual context, but they still do not think about the context on how to teach word problem

solving to their students in other forms of representations. Based on it, pre-service teachers need to improve their understanding related to content knowledge. According to Tripathi (2008), the use of some representations in teaching mathematics has been just like using a strong tool to make mathematics easier to understand for students. Moreover, the use of some representations also strengthens the students' understanding of learning about how to form and solve problems in mathematics.

Based on the discussion above, the thing that has to be known by teachers or pre-service teachers is by realizing the importance of using multiple representations in mathematics learning, that will increase the understanding of a teacher about the concept they taught, as disclosed by Durkaya et al (2011), multiple representations will help pre-service teachers to have an in-depth understanding of the concept. Because of that, it is suggested for the educators of mathematics pre-service teachers to apply multiple representations in class learning so that later when they become teachers, they can give multiple representations in different perspectives to accommodate different students' learning styles.

Thus, this study can be used as references to the teacher education program. Next, it can be used as a recommendation for continuous professional development for pre-service teachers to be a professional teacher. Where in the future pre-service teachers have comprehensive knowledge about mathematics and understand how to teach mathematics (Turnuklu & Yesildere, 2007).

CONCLUSION

Ideally, teachers can use multiple representations in mathematics learning, not only for their own but also can employ into their teaching. The use of multiple representations in mathematics learning could help students understanding mathematics concept better so that the students will be easy to solve mathematical problem appropriately. In addition, multiple representations have a function to accommodate different learning style.

The findings of this research are pre-service teachers tend to use symbols and verbal representation in word problem solving for themselves. Only a few of them use a combination of graphical representation. The other finding is pre-service teachers calculated word problem solving without fully realize and understand what is needed to be answered related to the question using multiple representations, even though, as a pre-service teacher, he/she has to know what he/she has to do in order to help students understanding word problem-solving. They tend to solve the questions as a subject of a research rather than as a teacher.

Here, the author invites teachers or education practitioners to pay more attention about giving experience using various representational in the class and strengthen the representational knowledge to the pre-service teacher since having multiple representation competencies is become one of the mathematics learning goals.

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