

Exploring student's mathematical argument in solving modeling solid with curved surface problem

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Abstract

This study aims to describe the mathematical argument of students in modeling mathematical problems with curved side shapes according to the classification of high and low initial mathematical abilities. This study used the descriptive method with the qualitative approach. Six ninth-grade students from junior high school in Surabaya were given five initial mathematics ability test questions. From the results of the initial mathematics ability test, two subjects were selected with the highest and lowest scores. Data instruments in this study use modeling solid with curved surface problem tests and interviews. The collected data were analyzed based on elements of mathematical argument, that is, claims, evidence, reasoning, and rebuttal. The result shows that subjects with high initial ability can make accurate claims, provide appropriate evidence and reason, and provide evidence to reject the counterclaim. This subject is categorized in level 2 of mathematical argumentation. While the subject with low initial ability cannot make claims accurately, the evidence and reason are unsuitable to support the claim and do not provide evidence and reasons to reject the counterclaim. This subject is categorized in level 1 of mathematical argumentation.

Keywords: modeling problem, solid with curved side, mathematical argumentation

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INTRODUCTION

Mathematical modeling is one way to represent complex problems in mathematical form (Ndi, 2018). Modeling means understanding a realistic problem, setting up a model of the problem and finding a solution by working on the model mathematically (Maaß, 2010). From this definition, the meaning is obtained that modeling becomes a bridge between complex real problems and the abstraction of mathematics. Thus, the model of mathematics created must be able to represent and explain the existing problem. In modeling, students will be trained to understand problems, connect mathematical concept and ideas to modeling, find the valid mathematical model, then solve the mathematical model found (Mayasari et.al., 2023).

In learning process, students are asked to formulate a problem, make early claims, construct mathematical models, test models, and solve and solve problems. From formulating assumptions to solving problems, students need to present mathematical arguments to strengthen whether the model created and the solutions obtained are appropriate to the context of the problem. The ability to argue involves the ability to express a (critical) reason accompanied by adequate data and theoretical support from a (logical) mathematical problem (Soekisno, 2015). Mathematical argumentation is an essential part of learning how to solve most types of problems and a powerful method for assessing ability to solve problems (Soekisno, 2015).

One of the studies that discuss the identification of arguments is the study put forward by Stephen E. Toulmin in Mahdiyyah & Susannah's research (2022). Data, claims, warrants, backing, qualifiers, and rebuttal are indicators in analyzing student's mathematical arguments using Toulmin's theory. Katherine McNeill and Joseph Krajcik simplified Toulmin's argumentation components into four components, namely claims, evidence, reasoning, and rebuttal. Ufairah (2022) describe those components, that is (1) Claim, where students are able to provide statements to answer the problems given; (2) Evidence, where students are able to show data that can support the statements put forward; (3) Reasoning, where Students are able to give reasons as

justification for statements accompanied by evidence; (4) Rebuttal, where students are able to reject statements and explain the conditions under which these statements do not apply.

In Mathematics, one of the complex mathematical concepts is found in the geometric material of curved side shapes. Curved side shapes are a type of geometric shapes that have special properties and high complexity. Besides having curved sides that are difficult to understand, curved side shapes also have complex shapes and are difficult to visualize in the form of drawings or models. Consequently, students need to make a mathematical representation that involves using mathematical concepts to describe and understand the geometrical properties and relationships in these spatial figures. According to Yanto (2017), mathematical modeling is the process of changing a problem or situation in the real world into a mathematical representation. Therefore, it is necessary to understand the concepts associated with the mathematical model that can represent the mathematical problem or situation.

An argument can be defined as a statement with justification and the opinion of a conclusion supported by a reason (Hidayat et al., 2018). Mathematical argumentation serves as a powerful tool for students to reason, justify, and communicate their mathematical ideas and claims. Students engage in mathematical argumentation to present logical and coherent reasoning supporting their mathematical models when dealing with the modeling of curved side shapes. Through argumentation, students can provide evidence and explanations for their chosen mathematical representations, showcasing their understanding of the underlying concepts and their ability to apply them effectively. Students can accurately demonstrate their understanding of the complex concepts associated with curved side shapes and create mathematical representations through argumentation.

A relevant study with the title Arguments Constructed within The Mathematical Modelling Cycle conducted by Dede (2018) found important findings that the relationship between argumentation and modeling can inspire students' arguments to reveal the difficulties that students may experience during the modeling process. Then the study by Herman et al. (2017), namely The Students' Mathematical Argumentation in Geometry, shows that in the justification of a statement, students use claims as a starting point. They elaborate on data by connecting on the mathematical object in question and then make a reason that supports against claims filed. They support their argument using inductive, algebra, visual, and perceptual.

The novelty of this study is that it is still rare to find research that considers the relationship between mathematical modeling and mathematical argumentation. Most do not directly refer to the components of mathematical argumentation. In addition, there is still a connection with the level of students' initial mathematical ability. This study focuses on investigating how students' mathematical argument in solving modeling solid with curved surface problems on initial mathematics ability. Similar research was also conducted by Alsina, Á. and Salgado, M. (2022) with the title Understanding Early Mathematical Modeling: First Steps in the Process of Translation Between Real World Contexts and Mathematics shows that children relate problem content to their prior knowledge. The modeling problem is used to see how students' mathematical arguments use Katherine McNeill and Joseph Krajcik's components.

METHOD

Subject of Research

This study used a qualitative descriptive research approach in which the researcher attempted to describe students' argument on modeling solid with curved surface problems. Six ninth-grade students from junior high school in Surabaya who had different initial abilities in mathematics were given five initial ability test questions. Initial ability test results were categorized based on test scores in low and high categories. The determination of categories is based on the minimum completeness criteria that apply in the school, where the high category (score > 80) and the low category (score < 60). The subject is selected based on students with low and high initial mathematics abilities who may face different challenges in understanding and solving mathematical modeling problems. By choosing this subject, the writer can analyze the difficulties faced by each group and present effective strategies to help students overcome these obstacles.

From the initial ability test result, there are two subjects representing each category of high and low initial ability. In addition, the subject selection also considers students' mathematics scores at school to ensure that the selected subject can represent the initial ability level of each category. The data obtained were then analyzed using qualitative analysis techniques consisting of data collection, data reduction, presentation of data in the form of test results and interviews, and drawing conclusions.

Instrument And Procedures

These three subjects will be given three solid with curved surface problems based on modeling math problems different from the initial ability test questions (Figure 1). Cahyono (2013) explains that mathematical modeling is the process of deriving a mathematical model of a phenomenon based on the assumptions used. Students are asked to organize and identify the mathematical aspects contained in the problem. Students are also given complete freedom to describe, simplify, interpret, and solve modeling problems in their way based on experience or prior knowledge they already have (Suwarsono, 2018). Students are given a 45-minute test with the condition that they may not use any references in answering the problem. Then, the interview will be conducted with each subject to find out their mathematical argument for solving the modeling problems. The four elements are claims, evidence, reasoning, and rebuttal.



Dalam rangka merayakan Hari Kemerdekaan di desanya, Wendy akan melukis 7 bola beton identik dengan jari-jari 20 cm. Ia membeli seember cat kuning berisi 2,5 liter. Namun, dia bingung apakah cat itu cukup untuk dia gunakan untuk mengecat seluruh bola beton. Bantu Wendy mencari tahu apakah cat yang dibelinya cukup untuk melapisi 7 permukaan bola beton yang identik atau tidak? (Diketahui bahwa 600 ml cat dapat melapisi 1 m² permukaan) Jelaskan temuan Anda!



In order to celebrate Independence Day in her village, Wendy will paint 7 identical concrete balls with a radius of 20 cm. He bought a bucket of yellow paint containing 2.5 liters. However, he was confused whether or not the paint was enough for him to use to paint the entire concrete ball. Help Wendy find out if the paint she bought is enough to coat the 7 identical concrete ball surfaces? (It is known that 600 ml of paint can cover 1 m² of surface) Explain your findings!

Figure 1. Test Instrument

The problem above is used to determine students' mathematical argumentation abilities in solving modeling problems involving the surface area of a solid with a curved surface. The problem asks students to determine whether the paint used is sufficient to cover the entire surface of the sphere by comparing the volume of paint needed and the volume of paint purchased. The subject must answer the problem based on the steps to solving mathematical modeling problems, that is, understanding the problem, formulating the problem, gathering data, analyzing the model, developing solution strategies, solving the model, validating the solution, and interpreting the results.

Then, interviews were conducted to find out how the student's argumentation process was in solving modeling problems using elements of mathematical argument by Katherine McNeill dan Joseph Krajcik. The two subjects will be interviewed alternately to find out the elements of each subject's arguments from claims, evidence, and reasoning. After that, both of them will be interviewed simultaneously to find out the elements of rebuttal if the two subjects have different arguments. If not, the two subjects will be asked to exchange ideas about the most appropriate. The following is a table of interview guidelines that will be used. This interview guide is flexible and can be changed according to the circumstances and conditions.

Data Analysis

The data obtained were then analyzed using qualitative analysis techniques consisting of data collection by looking for some reference articles, data reduction by selecting and eliminating irrelevant data in the interview results, presentation of data in the form of test results and interviews, and drawing conclusions. From the interview session, the mathematical argument of each subject will be known in each high and low category. After that, each element of the mathematical argument will be described into claims, evidence, reasoning, and rebuttal. Then, the results of student tests and interviews will be categorized based on the level of students' mathematical argumentation, according to Ufairah (2022):

Table 1. Level of Mathematical Argumentation

Level	Components of Mathematical Argumentation			
	Claims	Evidence	Reasoning	Rebuttal
0	Do not make claims or make inaccurate claims	Does not provide evidence, provides evidence but is not suitable to support the claim	Not giving reasons. Gives reasons but cannot link evidence to claims	Do not know the counterclaim and do not provide evidence and reasons to reject the counterclaim
1	Making incomplete claims	Provides evidence that is suitable but not sufficient to support the claim	Reiterate evidence and provide reasons but not sufficient to link claims to evidence	Knowing the counterclaim but not providing sufficient evidence and reasons to reject the counterclaim
2	Make accurate and complete claims	Provide sufficient and appropriate evidence to support the claim	Provide appropriate and sufficient reasons to link claims and evidence	Know the counterclaim and provide evidence and reasons to reject it

The following is the interview flowchart of student argumentation analysis in solving the given modeling problem.

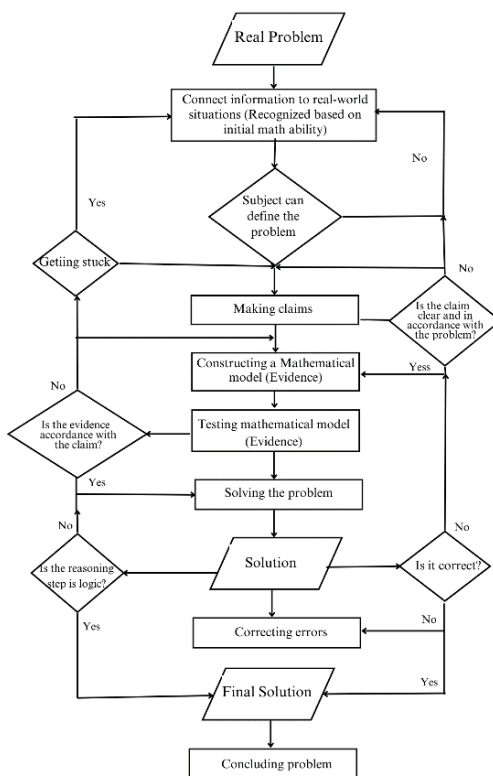


Figure 2. Flowchart of Analysis Student’s Mathematics Argument in Modeling Problem

RESULT AND DISCUSSION

Based on the results of the initial ability test obtained there are 2 subjects representing each category of high initial ability (HC) and low initial ability (LC). The following research results show the answer of each student solving modeling problem about the surface area of sphere. The discussion will describe how mathematical arguments of those two subject in solving modeling problems which include claims, evidence, reasoning, and rebuttal. The rebuttal was set aside because the claims made by the two subject is different.

Result

Claims, Evidence and Reasoning Low Category (LC)

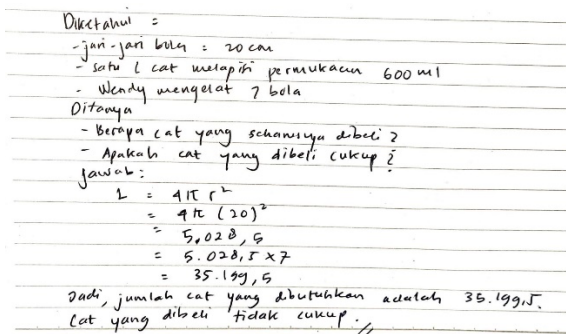


Figure 1. Subject with Low Category Answer

Subject with low initial ability cannot solve the problem well. Following are the results of interviews with the subject:

- R : “What do you understand based on the problem given? Have you solved a problem like this before?”
- LC : “Never.”

- R : "Well, then how did you find out what the problem being asked?"
- LC : "Because the problem relates to painting the surface of a sphere, I think that this problem leads to finding the surface area of all sphere and then comparing it to the paint that has been purchased."
- R : "What is your claim about the problem?"
- LC : "The paint that Wendy bought wasn't enough."
- R : "How do you get an estimate of the amount of paint needed to paint the entire surface of the spheres?"
- LC : "Formula for the surface area of a sphere is $4\pi r^2$, substitute a radius 20 cm to get a result 5,028.5 because there are seven spheres so the total surface area is 35,199.5. So, the paint that Wendy bought wasn't enough."

From the interview, it appears that the LC subject was unable to express the correct claim and the reasons given were not quite right. This is because the subject does not understand the information provided as seen by the absence of data processing *every 0.6 l can coat 1m²*. In addition, the calculations made are also inaccurate. It can be seen that the subjects did not convert cm to meters and ml to liters.

R : "Why are you sure that your answer is right? Is there something missing?"

LC : "No I think my answer is correct because I do according to what is asked."

Based on the interview result above, these are the components of mathematical argument of LC subject

Claims : The problem leads to finding the surface area of all sphere and then comparing it to the paint that has been purchased. Then the paint was not enough.

Evidence : Because the problem relates to painting the surface of a sphere. With calculating the surface area of seven sphere then comparing with the paint purchased can find the paint was not enough.

Reasoning : Using the surface area of a sphere formula that is $4\pi r^2$ with the radius 20 cm to find all seven sphere.

From these results, it was found that the LC subject was unable to formulate problems, model problems, solve problems, and draw conclusions. The LC subject's mathematical argument is not valid in claims, evidence, and reason.

Claims, Evidence and Reasoning High Category (HC)

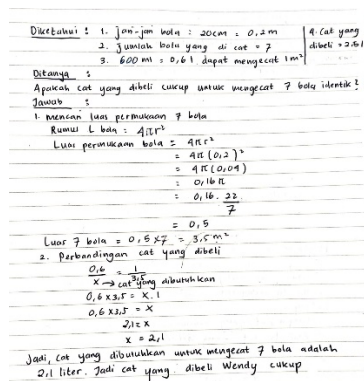


Figure 2. Subject with High Category Answer

Subjects with high initial ability can solve the problems given. Following are the results of interviews with the subject:

R : "What do you understand based on the problem given? Have you solved a problem like this before?"

- HC : "Never. What I understand is that there are seven spheres with a radius of 20 cm. They will then be painted. Then we will find out whether the paint purchased is sufficient to paint the surface of the entire sphere or not."
- R : "Well, if you have understood, can you explain the main claim that arises when you are solving the problem?"
- HC : "I think that I must find out the area of sphere and the paint is sufficient."
- R : "Why did you think like that? "
- HC : "I think the problem ask me to find out the area of sphere because when we paint something it's mean we should cover the surface. So that, I used the formula area of sphere that is $4\pi r^2$. Then I use proportion to find out paint needed with the given statement *every 0.6 l can coat $1m^2$* ."
- R : "Then, how did you get an estimate of the amount of paint needed to paint the entire surface of the spheres?"
- HC : "I calculated using the surface area of a sphere that is $4\pi r^2$ with a radius of 20 cm, the result is $0.5 m^2$. Then multiplied by seven the result is 3.5. Because every 1 liter of paint can coat 0.6 liters, the amount of paint needed is $3.5 m^2$ multiplied by 600 ml, the result is that the paint needed is 2.1 liters. Whereas, the paint purchased is 2.5 liters. So, the paint that Wendy bought was enough."

From the interview, HC subject is able to make a claims, evidence, and reasoning clearly and correctly. Based on the interview result above, these are the components of mathematical argument HC subject :

Claims : Find out the area of sphere. Then the paint is sufficient to cover all the sphere surface.

Evidence : Because when we paint something it's mean we should cover the surface area. So that, using the surface area of a sphere formula that is $4\pi r^2$.

Reasoning : Find out the total of area surface of seven sphere with the radius 20 cm then use proportion with the given statement *every 0.6 l can coat $1m^2$* to find the paint needed.

From these results, it was found that the HC subject was able to formulate problems, model problems, solve problems, and draw conclusions. The HC subject's mathematical argument is quite valid starting from claims, evidence, and reasons.

Rebuttal

Because the two subjects have different claims, then both of them will be interviewed simultaneously to see how their rebuttal component is. Both subjects are welcome to see each other's answers and express their respective opinions.

- T : "You two have a different claim. Now, take a look in your friend's answer. What do you think?"
- HC : "I think his answer is incorrect because he (LC) did not convert units and has not compared with the information the volume of paint needed to paint 1 m. Therefore, his answer is a very large number. Actually if logically this is not possible
- T : "Well, how about you (LC)?"
- LC : " I agree with her (HC)."

Based on the interview, HC subject conveyed the valid reason for disagreeing with LC answer. HC refute or reject a conclusion that the specific condition is not a description of the general condition. According to Lamena et al., (2018) when finding counter example students with high ability to perform systemization (compiling in deductive evidence) as a condition of proof, on the contrary. Whereas, LC is agree with HC answer. So, HC is know the counterclaim and provide evidence and reasons to reject it while LC not provide any rejection.

Discussion

From the interview results, it was found that the LC subject was unable to model the problem because there was a miscalculation and misunderstanding. Invalid claims may become valid if students are aware of their mistakes and restructure (Laamena et al., 2018). Because of this, the claims put forward are inaccurate; namely, the paint purchased is not enough to coat the seven surfaces of the ball. The LC subject has been able to claim that the given problem will utilize the use of the spherical surface area formula. However, something important was missed in the calculation, namely the unit conversion and paint volume that can be used for a certain surface area. The LC subject was sure of the answers he got, so he didn't notice that something was missed. Therefore, the evidence and reason components presented are not quite right. In the rebuttal component, the LC subject cannot express rejection of answers that are contrary to his own. It can be seen that the subject of LC has a level of mathematical argumentation at level 1 because the claims made are not appropriate. Hence the evidence and reasons they have are also inaccurate. In addition, in the rebuttal component, he was unable to provide refutations and evidence for opinions that conflicted with his own.

Whereas, HC subjects were able to convey their arguments validly. Starting from the claim that he conveyed was valid, that is, the paint that was purchased was sufficient. Then put forward evidence that supports that the problem is solved by using the formula for the surface area of a sphere. The calculations are also correct. Therefore the reasons put forward are also valid. In the rebuttal component, it appears that the HC subject is able to present valid and sufficient evidence to refute answers that contradict his own. It can be seen that the HC subject has a high level of mathematical argumentation, namely at level 2 because the claims submitted are appropriate and the evidence and reasons presented are also sufficient and appropriate to support the claims. In addition, it can also convey evidence of rejection properly.

The results of the discussion will be summarized in the form of a table below:

Table 2. The Result of each Subject

Subject	Components of Mathematical Argumentation				Level
	Claims	Evidence	Reasoning	Rebuttal	
HC	Make accurate and complete claims	Provide sufficient and appropriate evidence to support the claim	Provide appropriate and sufficient reasons to link claims and evidence	Know the counterclaim and provide evidence and reasons to reject it	2
LC	Making inaccurate claims	Provides evidence but is not suitable to support the claim	.Gives reasons but cannot link evidence to claims	Do not know the counterclaim and do not provide evidence and reasons to reject the counterclaim	1

CONCLUSION

The research results obtained show that: subject with High Initial Ability can formulate problems, construct models, complete models, validate models, and conclude problems correctly. From mathematical argumentation components, subject with High Initial Ability can make accurate claims, provide appropriate evidence and reason to support the claim, and provide evidence to reject the counterclaim. This subject is categorized in level 2 of mathematical argumentation. So that, the aspect of mathematical argumentation show validation of solving modeling problems properly. Students with Low Initial Ability can formulate problems, construct models, complete models, validate models, and conclude but with errors in understanding information and calculation errors. From mathematical argumentation components, subject with Low Initial Ability cannot make claims

accurately, provides evidence and reason but is not suitable for support the claim provide appropriate evidence and reason to support the claim, and do not provide evidence and reasons to reject the counterclaim. This subject is categorized in level 1 of mathematical argumentation. In the aspect of mathematical argumentation, it is less able to show validation of solving modeling problems properly. The research is limited by explaining how students' arguments in solving modeling problems are based on low and high levels of initial math skills. Future research can expand the scope of the selected subjects. This can be viewed in terms of gender, AQ, and others or at a higher level. By involving a more diverse sample, research can provide a more comprehensive understanding of students' mathematical arguments in the context of modeling.

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