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Modified Newtonian Dynamics (MOND) vs Newtonian Dynamics: The Simple Test to Solve the Constant Speed of Galaxy Rotation

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Abstract
The rotation curve of galaxies for M33 in 1959 by Louise Volders gave the new hypothetical about the invisible matter that contributes inside of the galaxy, which later we call dark matter (DM). However, recently the theory about DM is still incomplete to understand this matter. This situation makes some scientists look for alternative ways such as *f*(R) gravity and conformal gravity theory. We have studied Modified Newtonian Dynamics (MOND) and Newtonian Dynamics (ND). We try to show the simple model that aims to give an analysis that MOND can correct to solve the constant speed of galaxy rotation. For simplicity, we consider the value of $\alpha = 1$. The graph shows that the MOND model has a constant speed of 100 kilometres per second. While for the ND model, the speed will decrease for radius goes to infinity because the speed is dependent on r. Based on this result, we obtain that MOND can show the constant speed of galaxy rotation than ND. This result can conclude that MOND can solve the rotation curve of the galaxy.
Keywords: Modified Newtonian Dynamics (MOND), Newtonian Dynamics (ND), galaxy rotation curve.

1. Introduction

The fact about the rotation curve of galaxies for M33 in 1959 by Louise Volders opened the gate of interest in astronomy [1], [2]. That research appears the new hypothetical about the invisible matter that contributes inside of the galaxy. If we focus on the large scale, however, the grand theory known as general relativity fails to account for the dynamic of galaxy rotation [3], [4]. There should be any unless a massive component of unknown matter, which we later call dark matter (DM) [5]–[7].

On the other hand, the theory about DM is still incomplete in the current condition [8]–[10]. We don't have enough information about this matter. This situation makes some scientists look for alternative ways such as *f*(R) gravity [5], [10], [11] and conformal gravity theory [2], [9], [12]–[14]. Then various formulations of the modified Newtonian dynamics (MOND) paradigm [15]–[17], tensor-vector-scalar theory (TeVeS) [18] and quasi-linear MOND [19] also as an alternative way to solve that problem.

The last topic mentioned above also has a historical value that is motivated by the rotation curve galaxy condition [14], [20], [21]. The main reason was based on the calculation of galaxy speed where Newtonian Dynamics (ND) gave the result that did not match the observed data [22]. The constant speed of the galaxy, if we compared with Kepler's law as an implementation of object rotation, has different [20], [22]. The object should have a speed that decreases at radius toward far away [22]. Indeed, this fact may be concluded that ND has successfully been explaining and giving a good prediction for regular scale. But on a large scale, it is stuck and any problem with actual observation [23]–[25].

Therefore, we go back to M. Milgrom proposed theory and his recent observation [15], [26]–[28]. MOND appears to be trying to answer the problem about the constant speed of the galaxy. Many researchers have shown that MOND can be considered the new model of modifying gravity that is able with observation [16], [17], [24], [25], [29]–[32]. However, almost all of their work needs complicated tools and tight analysis. So, in this research, we will work on a simple test that shows the differentiating of the MOND and ND models to solve the galaxy rotation problem.

2. Method

2.1. Mathematic of ND and MOND

We have been familiar with ND law which works well for systems. Generally, we can write as

$$\vec{F} = m\vec{a} \tag{1}$$

where $\vec{F}$ is the force that works on the system, m for the object mass, and $\vec{a}$ is an acceleration of object motions. Then, for universal gravity force, we have

$$\vec{F} = G\frac{mM}{\vec{r}^2} \tag{2}$$

where G is the gravitational constant, M is the mass of another object, and $\vec{r}$ is the distance between the centres of the two objects. Then, we can write the acceleration

$$\vec{a} = G\frac{M}{\vec{r}^2} \tag{3}$$

Then, we have a relation between acceleration and velocity $\vec{v}$ of a circular orbit depending on the object's radius. Therefore, we necessary to write as

$$\vec{a} = \frac{\vec{v}^2}{\vec{r}} \tag{4}$$

thus, we obtain velocity as

$$\vec{v}^2 = \frac{GM}{\vec{r}} \tag{5}$$

and for calculating speed, we have to take the modulus of velocity so we can write

$$|v| = \sqrt{\frac{GM}{|r|}} \tag{6}$$

How is the MOND model? For the MOND model, we give the additional function of Newton's law on Equation 1 as follows

$$\vec{F} = m\mu\left(\frac{\vec{a}}{\alpha}\right)\vec{a} \tag{7}$$

where $\mu(s)$ is an additional function for modifying Newtonian model, and $s = \vec{a}/\alpha$ for $\alpha \cong 1.2 \times 10^{-8}$ cm/s^2 as a small constant of acceleration [33], [34]. If function $\mu(s)$ satisfies asymptotic behaviour, which means $\mu = 1$ for $\vec{a} >> \alpha$, and for the strong-field regime or MOND model regime, we have $\mu = \vec{a}/\alpha$ for $\vec{a} << \alpha$. Thus,

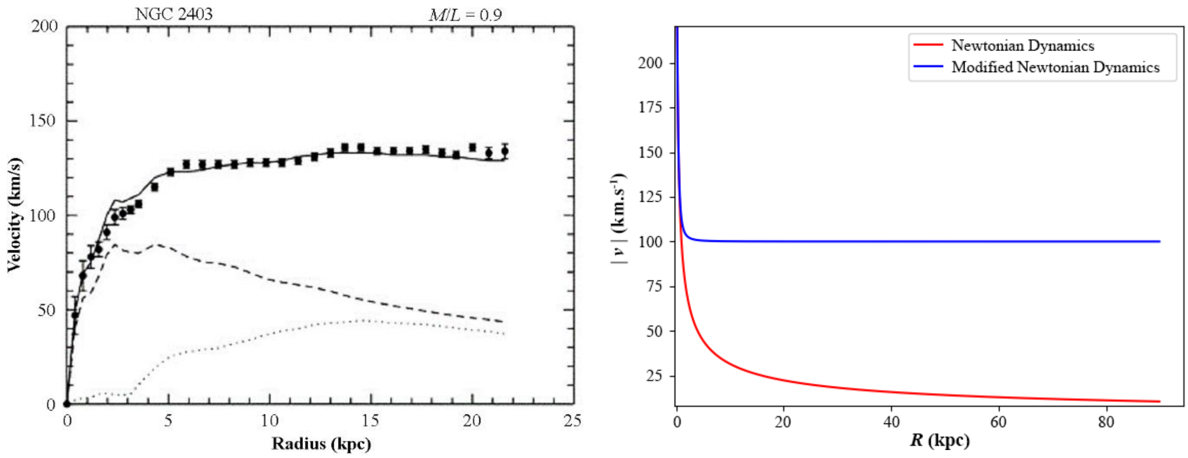
$$\frac{\vec{F}}{m} = \frac{\vec{a}^2}{\alpha} \tag{8}$$

We can use the relation on Equation 4, and we obtain

$$|v|^4 = GM\alpha \tag{9}$$

Finally, we have the speed of the MOND model as

$$|v| = \sqrt[4]{GM\alpha} \tag{10}$$



(a) (b)

Figure 1. (a) Observation of Andromeda Galaxy [35] and (b) The simple test plot of ND and MOND model.

3. Result and Discussion

In the previous part, we have successfully shown the mathematics of the MOND and ND models. Then, to analyze the equation package, we need to show how the differentiation of each model. We show the simple test of the MOND and ND models to solve the constant speed of galaxy rotation. Suppose $\alpha = 1$ then the speed of ND and MOND models, in Equation 6 and 10 can be shown in Figure 1.

Figure 1a is the result of Andromeda observation, in which the point curve is the speed of Andromeda rotation [35]. The other curves are obtained from the calculations of some models. Which is the ND model shown as dash-line. The curve of the ND model has decreased for radius going to infinity. Then dot-line curve has obtained based on the dark matter model just for the bulge galaxy. So, it indicates that the model is not fully calculated alongside the galaxy. Then, the solid curve has obtained from the MOND model. This plot has been obtained based on great observation. It means that we need complicated tools and expensive costs. Therefore, we try to show the simple model that aims to give an analysis that MOND can correct to solve the constant speed of galaxy rotation. Like in Figure 1b, we consider for simplicity the value of $\alpha = 1$. The plot shows that the MOND model has a constant speed of 100 km/s. While for the ND model, because the speed depends on r, which means the radius of the galaxy, the speed will decrease for radius goes to infinity.

This result gave information that the MOND model can solve the constant speed of the galaxy without necessary give additional strange matter. So, dark matter may be the potential to answer many problems in the mystery of the universe. Nevertheless, the reality and information of dark matter that we do not know yet now. Then, the MOND model appears as an alternative way to solve this problem without necessarily explaining dark matter.

4. Conclusion

The MOND model can solve the problem of rotation curves galaxy or constant galaxy rotation speed. As we can see in Figure 1b, this plot shows that the MOND model obtained a constant speed than the ND model prediction. The MOND model has a constant speed of 100 km/s. While for the ND model, because the speed depends on r, which means the radius of the galaxy, the speed will decrease for radius goes to infinity. As a simple test, this research can be considered for another observation in the future that has similar cases.

References

[1] J. A. Sellwood, "Secular evolution in disk galaxies," *Rev. Mod. Phys.*, vol. 86, no. 1, p. 1, 2014, doi: 10.1103/RevModPhys.86.1.

[2] T. Naab and J. P. Ostriker, "Theoretical challenges in galaxy formation," *Annu. Rev. Astron. Astrophys.*, vol. 55, pp. 59–109, 2017, doi: 10.1146/annurev-astro-081913-040019.

[3] P. Galianni, M. Feix, H. Zhao, and K. Horne, "Testing quasilinear modified newtonian dynamics in the solar system," *Phys. Rev. D*, vol. 86, no. 4, p. 044002, 2012, doi: 10.1103/PhysRevD.86.044002.

[4] B. Joachimi *et al.*, "Galaxy alignments: An overview," *Space Sci. Rev.*, vol. 193, no. 1, pp. 1–65, 2015, doi: 10.1007/s11214-015-0177-4.
[5] R. Zaregonbadi, M. Farhoudi, and N. Riazi, "Dark matter from f (R, T) gravity," *Phys. Rev. D*, vol. 94, no. 8, p. 084052, 2016, doi: 10.1103/PhysRevD.94.084052.
[6] J. I. Read, "The local dark matter density," *J. Phys. G: Nucl. Part. Phys.*, vol. 41, no. 6, p. 063101, 2014, doi: 10.1088/0954-3899/41/6/063101.
[7] G. Bertone and T. M. Tait, "A new era in the search for dark matter," *Nature*, vol. 562, no. 7725, pp. 51–56, 2018, doi: 10.1038/s41586-018-0542-z.
[8] G. Hinshaw *et al.*, "Nine-year wilkinson microwave anisotropy probe (WMAP) observations: Cosmological parameter results," *Astrophys. J. Suppl. Ser.*, vol. 208, no. 2, p. 19, 2013, doi: 10.1088/0067-0049/208/2/19.
[9] D. Baumann and L. McAllister, *Inflation and String Theory*. Cambridge, United Kingdom: Cambridge University Press, 2015.
[10] A. Joyce, B. Jain, J. Khoury, and M. Trodden, "Beyond the cosmological standard model," *Phys. Rep.*, vol. 568, pp. 1–98, 2015, doi: 10.1016/j.physrep.2014.12.002.
[11] G. Leon and E. N. Saridakis, "Dynamical behavior in mimetic F (R) gravity," *J. Cosmol. Astropart. Phys.*, vol. 2015, no. 04, p. 031, 2015, doi: 10.1088/1475-7516/2015/04/031.
[12] K. Koyama, "Cosmological tests of modified gravity," *Rep. Prog. Phys.*, vol. 79, no. 4, p. 046902, 2016, doi: 10.1088/0034-4885/79/4/046902.
[13] R. Myrzakulov, L. Sebastiani, S. Vagnozzi, and S. Zerbini, "Static spherically symmetric solutions in mimetic gravity: Rotation curves and wormholes," *Class. Quantum Gravity*, vol. 33, no. 12, p. 125005, 2016, doi: 10.1088/0264-9381/33/12/125005.
[14] X. Wu and P. Kroupa, "Galactic rotation curves, the baryon-to-dark-halo-mass relation and space–time scale invariance," *Mon. Notices Royal Astron. Soc.*, vol. 446, no. 1, pp. 330–344, 2015, doi: 10.1093/mnras/stu2099.
[15] M. Milgrom, "MOND theory," *Can. J. Phys.*, vol. 93, no. 2, pp. 107–118, 2014, doi: 10.1139/cjp-2014-0211.
[16] A. O. Hodson and H. Zhao, "Generalizing MOND to explain the missing mass in galaxy clusters," *Astron. Astrophys.*, vol. 598, p. A127, 2017, doi: 10.1051/0004-6361/201629358.
[17] S. Mendoza, "MOND as the basis for an extended theory of gravity," *Can. J. Phys.*, vol. 93, no. 2, pp. 217–231, 2015, doi: 10.1139/cjp-2014-0208.
[18] E. Barrientos and S. Mendoza, "A relativistic description of MOND using the Palatini formalism in an extended metric theory of gravity," *Eur. Phys. J. Plus*, vol. 131, no. 10, pp. 1–10, 2016, doi: 10.1140/epjp/i2016-16367-0.
[19] A. Karam, L. Marzola, T. Pappas, A. Racioppi, and K. Tamvakis, "Constant-roll (quasi-) linear inflation," *J. Cosmol. Astropart. Phys.*, vol. 2018, no. 05, p. 011, 2018, doi: 10.1088/1475-7516/2018/05/011.
[20] Y. Sofue, "Rotation and mass in the Milky Way and spiral galaxies," *Publ. Astron. Soc. Jpn.*, vol. 69, no. 1, p. R1, 2017, doi: 10.1093/pasj/psw103.
[21] Y. Sofue, "Rotation curve of the Milky Way and the dark matter density," *Galaxies*, vol. 8, no. 2, p. 37, 2020, doi: 10.3390/galaxies8020037.
[22] M. Cappellari, "Structure and kinematics of early-type galaxies from integral field spectroscopy," *Annu. Rev. Astron. Astrophys.*, vol. 54, pp. 597–665, 2016, doi: 10.1146/annurev-astro-082214-122432.
[23] P. Bull *et al.*, "Beyond ΛCDM: Problems, solutions, and the road ahead," *Phys. Dark Universe*, vol. 12, pp. 56–99, 2016, doi: 10.1016/j.dark.2016.02.001.
[24] J. Schee, Z. Stuchlík, and M. Petrásek, "Influence of the cosmic repulsion on the MOND model of the Magellanic Cloud motion in the field of Milky Way," *J. Cosmol. Astropart. Phys.*, vol. 2013, no. 12, p. 026, 2013, doi: 10.1088/1475-7516/2013/12/026.
[25] B. Famaey and S. S. McGaugh, "Modified newtonian dynamics (MOND): Observational phenomenology and relativistic extensions," *Living Rev. Relativ.*, vol. 15, no. 1, pp. 1–159, 2012, doi: 10.12942/lrr-2012-10.
[26] M. Milgrom, "Testing MOND over a wide acceleration range in x-ray ellipticals," *Phys. Rev. Lett.*, vol. 109, no. 13, p. 131101, 2012, doi: 10.1103/PhysRevLett.109.131101.

[27] M. Milgrom, "The MOND paradigm of modified dynamics," *Scholarpedia*, vol. 9, no. 6, p. 31410, 2014, doi: 10.4249/scholarpedia.31410.

[28] M. Milgrom, "Universal modified newtonian dynamics relation between the baryonic and "dynamical" central surface densities of disc galaxies," *Phys. Rev. Lett.*, vol. 117, no. 14, p. 141101, 2016, doi: 10.1103/PhysRevLett.117.141101.

[29] G. N. Candlish, R. Smith, and M. Fellhauer, "Numerical simulations of modified newtonian dynamics," *J. Phys.: Conf. Ser.*, vol. 720, no. 1, p. 012012, 2016, doi: 10.1088/1742-6596/720/1/012012.

[30] B. Famaey, S. McGaugh, and M. Milgrom, "MOND and the dynamics of NGC 1052− DF2," *Mon. Notices Royal Astron. Soc.*, vol. 480, no. 1, pp. 473–476, 2018, doi: 10.1093/mnras/sty1884.

[31] B. A. Ahmed, L. K. Abood, and M. A. Salih, "Simulation on rotation curve of spiral galaxies," *J. Phys.: Conf. Ser.*, vol. 1234, no. 1, p. 012031, 2019, doi: 10.1088/1742-6596/1234/1/012031.

[32] A. McDaniel, T. Jeltema, and S. Profumo, "X-ray shapes of elliptical galaxies and implications for self-interacting dark matter," *J. Cosmol. Astropart. Phys.*, vol. 2021, no. 05, p. 020, 2021, doi: 10.1088/1475-7516/2021/05/020.

[33] I. Banik *et al.*, "The global stability of M33 in MOND," *The Astrophys. J.*, vol. 905, no. 2, p. 135, 2020, doi: 10.3847/1538-4357/abc623.

[34] G. Pezzulli, F. Fraternali, and J. Binney, "The angular momentum of cosmological coronae and the inside-out growth of spiral galaxies," *Mon. Notices Royal Astron. Soc.*, vol. 467, no. 1, pp. 311–329, 2017, doi: 10.1093/mnras/stx029.

[35] P. E. M. Piña *et al.*, "Robust H I kinematics of gas-rich ultra-diffuse galaxies: Hints of a weak-feedback formation scenario," *Mon. Notices Royal Astron. Soc.*, vol. 495, no. 4, pp. 3636–3655, 2020, doi: 10.1093/mnras/staa1256.