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Destroying a Kerr Black Hole

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Abstract

Based on three fundamentals aspects presented by Penrose, Wald, and Hawking, the black hole's destroying scenario can be made. The main problem of this destroying process is about the presence of black hole's angular momentum and charge. Especially for this research which is only taking an action into a rotating uncharged black hole (Kerr Black Hole). This research showed that in order to destroy a Kerr Black Hole (especially a near-extremal Kerr Black Hole), one needs to remove its angular momentum instead by forcing it in extremal conditions and let Hawking Radiation occurs. On the other hand, the research showed that particle's mass is linearly dependant in Penrose Process. The result also showed (which ; is a black hole's angular momentum and is a black hole mass) cannot be reached because only black hole with negative net-energy can fulfil the condition. In another side, we calculated the minimum power needed by hole to kick the particles out.

Keywords: angular momentum, black hole's destroying scenario, Hawking Radiation, Kerr Black Hole, Penrose Process

Abstrak

Berdasarkan 3 aspek dasar yang ditunjukkan oleh Penrose, Wald, dan Hawking, skenario penghancuran lubang hitam dapat dilakukan. Masalah utama dalam proses penghancuran lubang hitam adalah keberadaan momentum sudut dan muatan listrik dalam lubang hitam. Dalam penelitian ini hanya dibahas mengenai lubang hitam berotasi tak bermuatan (lubang hitam Kerr). Hasil penelitian menunjukkan untuk menghancurkan lubang hitam berotasi, maka hal yang paling penting adalah menyingkirkan rotasi lubang hitam terlebih dahulu, disamping memaksanya menuju titik ekstrim, dan membiarkan Radiasi Hawking terjadi. Pada penelitian ini didapat pula bahwa kondisi tidak mungkin terpenuhi karena hanya lubang hitam dengan energi-net negatif yang dapat melakukannya. Didapat pula perhitungan tenaga yang diperlukan lubang hitam untuk menendang partikel keluar.

Kata Kunci: momentum sudut, skenario penghancuran lubang hitam, Radiasi Hawking, Lubang hitam Kerr, Proses Penrose.

1. Introduction

The effort which theoretically used to destroy a black hole has been discussed by [1], [2], [3], and [4]. The basic idea is putting test bodies with specific angular momentum and charge to make black hole pass its extremity. But, the method proved that no matter how many test bodies hit the black hole, there is a mechanism that black hole will kick those objects out. In this paper, Hawking radiation [5] and Penrose process [6] will be discussed along the Page's [7, 8] particle creation in the black holes.

2. Wald's Gedanken Experiments

In this section, Wald's (*gedanken*) experiments [1] to destroy a black hole will be reviewed. After all, this kind of experiments only happen in theoretical aspects. Wald's experiments were actually came from the basic idea that spin and charge are mutually the main aspects of black hole's perturbations. By simple logic, Wald wanted to crush the black hole by increasing those perturbations until they reach their extreme point $(Q^2 + a^2 = M^2)$. Wald stated that such particles can destroy a black hole if condition

$$E < (Qem + aL)/(M^2 + a^2) \tag{1}$$

can be met, while the convention $G = c = \hbar = k_B = 1$ are used for simplicity. With E, Q, and L are respectively representing energy, charge, and angular momentum of particle, hence m, e, and $a = \mathcal{J}/M$ are respectively representing energy, charge, and angular momentum of the hole. Since in this paper, the topics only limiting the case for a rotating black hole, Eq. (1) can be read:

$$E < (aL)/(M^2 + a^2)\,. \tag{2}$$

The next step will be simple, in order to destroy a Kerr black hole, Eq. (11) must be fulfilled. Before using the requirement of Eq. (11), Kerr metric need to be reinvestigated as follows

$$ds^2 = \frac{\Delta - a^2 \sin^2\theta}{\rho^2} dt^2 + \frac{4Mar\sin^2\theta}{\rho} dtd\phi - \frac{\rho^2}{\Delta} - \rho^2 d\theta^2 - \frac{\Sigma^2 \sin^2\theta}{\rho^2} d\phi^2 \tag{3}$$

with $\rho^2 = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - 2Mr + a^2$, and the last, for Σ can be translated via $\Sigma^2 = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta$. The signature $(+,-,-,-)$ has been adapted instead of signature used by its original paper [1].

Based on the metric (3), the Lagrangian can be formed as follows:

$$\mathcal{L} = \frac{1}{2} Mg_{\mu\nu} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} \tag{4}$$

The next step will be simple, by putting the Lagrangian and formed an Euler-Lagrange equations, one obtains

$$E = p_t = \frac{\partial\mathcal{L}}{\partial\dot{t}} = Mg_{t\nu}\frac{dx^\mu}{ds}, \tag{5}$$

and

$$-L = p_\phi = \frac{\mathcal{L}}{d\dot{\phi}} = Mg_{\phi\nu}\frac{dx^\mu}{ds}. \tag{6}$$

With both E and L are constant through affine parameters, because both $\partial/\partial\dot{t}$ and $\partial/\partial\dot{\phi}$ are Killing vectors. After all, ds^2 is defined as follows

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \rightarrow 1 = g_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = \frac{1}{M} g^{\mu\nu} p_\mu p_\nu\ . \tag{7}$$

By substitution of eq (4)-(6) into (7), defining μ and ν respectively, then one can arise their components and obtains

$$E = \frac{g^{t\phi}L \pm [(g^{t\phi})^2 L^2 - (g^{tt}g^{\phi\phi}L^2 g^{rr}p_r^2 + g^{\theta\theta}p_\theta^2 - M^2)^{1/2}}{g^{tt}} \tag{8}$$

Because the value inside the square bracket is positive, one may get the simple from as

$$E \geq \frac{g^{t\phi}L}{g^{tt}} = \frac{a(r^2 - a^2 - \Delta^2)L}{(r^2 + a^2)2 - a^2\Delta\sin^2\theta}. \tag{9}$$

When falling particles met the condition as $r = r_+ = m - (m^2 - a^2)1/2 = m$ and $\theta = 0$, one obtains

$$E \geq \frac{aL}{m^2 + a^2}\,. \tag{10}$$

In the presence of Eq. (10), it seems impossible to obtain Eq. (2). It means, in short, there is impossible to destroy a black hole by inserting a test particle. Wald [1] translated this phenomena by stated that black hole is repelling the test particle when it almost reach its extreme point.

3. Hawking Radiation

There is no doubt about the contribution of hawking radiation as the main source of destroying a black hole. In 1974-1975, Hawking [5] proposed that black holes are emitting particles continuously. This evaporation was based on consequence of his previous paper [9] on black hole thermodynamic.

Bardeen, Hawking, and Carter [9] proposed that thermodynamics may apply on black holes. In classical thermodynamics, it stated that

$$dM = TdS + \Omega d\mathcal{J} + \Phi dQ, \tag{11}$$

with M is total energy, T is temperature, S is entropy, Ω is angular velocity, $\mathcal{J}$ is angular momentum, Φ is electrostatic potential, and Q is charge. In another hand, the black hole thermodynamics can be translated from Eq. (11) as follows

$$dE = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ, \tag{12}$$

which κ is black hole's surface gravity and A is black hole's horizon. By adapting the concept of classical thermodynamics, one may recognize the formula of black hole's thermodynamics as

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{8M\pi}, \tag{13}$$

with T_H is Hawking temperature. By Eq. (13), the power radiates by non-rotating hole can be formulated by

$$P_{Schwarzchild} = -\sigma T_H^4 A, \tag{14}$$

where the minus sign can be considered by the energy loses by black hole, and $A = 16\pi M^2$. These two last equations only valid for Schwarzchild or non-rotating black hole. Since Hawking radiation is decreasing as mass of the hole increasing by factor powered by 4, it means supermassive black holes (with enormous mass) are getting more difficult to be destroyed.

In further case for rotating black holes or Kerr black holes, one may found that in those black holes will have horizon and surface gravity as follows

$$A = 4\pi([M + (M^2 - a^2)^{1/2}]^2 - a^2)$$

$$\kappa = \frac{(M^2 - a^2)^{1/2}}{2M[M + (M^2 - a^2)]^{1/2}}, \tag{15}$$

which Hawking temperature can be written by

$$T_H = \frac{(M^2 - a^2)^{1/2}}{4\pi M[M + (M^2 - a^2)]^{1/2}}. \tag{16}$$

One can formulates the amount power radiates by rotating black hole by the previous equations. Also, by putting $a < M$ for a slow-rotating black hole, one obtains

$$P_{Kerr} \approx \sigma \left[\frac{1}{4\pi}\right] \frac{\pi}{M^2} \left[1 - \frac{4a^2}{M^2}\right]. \tag{17}$$

By comparing eq. (14) and (17), it is clear that $P_{Kerr} < P_{Schwarzchild}$. In another case, one may regards, rotation keeps the black holes live longer. One conclusion arise, in order to destroy a rotating black hole, one needs to remove its angular momentum.

In additions of Hawking radiation which may occur in black hole, one can predict particles outcome from Page's calculation[7][8] in the form

$$\frac{d}{dt}\mathcal{J} = \sum \frac{1}{2\pi} \int \Gamma_{s\omega lmp} \left(\exp[2\pi\kappa^{-1}(\omega - M\Omega - e\Phi_c)] \mp 1 \right) M d\omega, \tag{18}$$

where $\Gamma_{s\omega lmp}$, Ω, and Φ_c are respectively grey factor, black hole's angular velocity, and electrostatic potential. The indices s, l, and m, are quantum numbers, while the last number of ω and p are respectively energy and polarization of particle(s). Eq. (18) indices that particles which emerged from the hole are, instead carrying black hole's energy, also draining black hole's angular momentum which is faster than its energy.

4. Penrose Process

There is another interesting fact which can only occur in Kerr black hole, namely Penrose process, which describes that black hole's M and $\mathcal{J}$ can be extracted via some conditions. One can find that black hole's energy can be extracted via Penrose process by relation

$$\delta \mathcal{J} < \frac{\delta M}{\Omega_H} \tag{19}$$

where $\mathcal{J}$, M , and Ω_H are respectively angular momentum, mass/energy, and angular velocity of the hole. In Eq. (19), there is a problem arise: efficiency of the Penrose process is far too small [10].

There are conditions when black hole repelling particle(s). The first condition occurs when a nearly-extremal black hole is repelling particles, which is described in the previous section. The second condition occurs when a black hole evaporates. The third condition occurs when Penrose process happened in a black hole. One may represent these conditions into a single meaning, namely *black hole kick* for simplicity which its definition is easily understandable.

In later discussion, one may calculate black hole's kicking power by deriving from Kerr metric as follows

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{4Ma}{r}dtd\phi - \frac{r^2}{\delta}dr^2 - \left(r^2 + a^2 + \frac{2Ma^2}{r}\right)d\phi^2, \tag{20}$$

where above equation is the simpler version of eq. (3) with $\theta = \pi / 2$. From this metric one obtains[1]

$$g^{tt} = \frac{1}{\Delta}\left(r^2 + a^2 + \frac{2Ma^2}{r}\right), \tag{21}$$

$$g^{rr} = -\frac{\Delta}{r^2}, \tag{22}$$

$$g^{t\phi} = \frac{2Ma}{r\Delta}, \tag{23}$$

and

$$g^{\phi\phi} = -\frac{1}{\Delta}\left(1 - \frac{2M}{r}\right). \tag{24}$$

By Lagrangian $\mathcal{L} = g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$, one can automatically obtain p_t and p_ϕ which can be described by

$$p_t = g_{tt}\dot{t} + g_{t\phi}\dot{\phi} = \left(1 - \frac{2M}{r}\right)\dot{t} + \frac{2Ma}{r}\dot{\phi} = k \tag{25}$$

and

$$p_\phi = g_{\phi t}\dot{t} + g_{\phi\phi}\dot{\phi} = \frac{2Ma}{r}\dot{t} - \left(r^2 + a^2 + \frac{2Ma^2}{r}\right)\dot{\phi} = -h, \tag{26}$$

where $\dot{t}$ is actually $dt / d\tau$ and $\dot{\phi}$ is $d\phi / d\tau$ (τ is proper time), which both of them are Killing vectors. In the next step, by writing it down

$$E / m = 1 = g^{\mu\nu} p_\mu p_\nu = g^{rr} p_t^2 + 2g^{t\mu} p_t p_\phi + g^{\phi phi} p_\phi^2 + g^{rr} p_r^2, \tag{27}$$

one can rearrange the result by substituting Eq. (21)-(24) as follows:

$$\frac{1}{2}\dot{r}^2 = \frac{k^2 - 1}{2} + \frac{M}{r} + \frac{a^2(k^2 - 1) - h^2}{2r^2} + \frac{M(h - ak)^2}{r^3}. \tag{28}$$

One can reinterpret this result in the left side as kinetic energy of the particle per unit mass, the first term in the right side is the total energy, and the remaining terms can be united as

$$V_{eff}(r,h,k) = -\frac{M}{r} - \frac{a^2(k^2 - 1) - h^2}{2r^2} - \frac{M(h - ak)^2}{r^3}, \tag{29}$$

[1] The derivation of Eq. (21)-(30) can be found in [11]

namely effective potential. One may see the first term of Eq. (29) as classical potential gravity.

Based on Eq. (28), there is a requirement that the particle is orbiting the hole, which $\dot{r} = 0$, since there is no radial velocity included for stable orbit. By rewriting Eq. (28) with right side is zero, one obtains

$$\frac{k^2-1}{2} = -\left(\frac{M}{r} + \frac{a^2(k^2-1)-h^2}{2r^2} + \frac{M(h-ak)^2}{r^3}\right) \tag{30}$$

or

$$\varepsilon = -\left(\frac{M}{r} + \frac{a^2(k^2-1)-h^2}{2r^2} + \frac{M(h-ak)^2}{r^3}\right) = V_{eff}(r,h,k)\,. \tag{31}$$

In that case, one can obtain the minimum kicking power provided by black hole. At least its kicking power should make the particles orbiting the hole. So one obtains the black hole's kicking power per unit particle's mass at least should be equal to Eq. (31) or

$$\varepsilon = \frac{k^2-1}{2}\ . \tag{32}$$

Later, by rewritten Eq. (19) with stolen rotational energy $\mathcal{J}_S$, and stolen total energy by M_S as follows

$$\Omega_H \mathcal{J}_S < M_S\,. \tag{33}$$

Since M_S equals $m\varepsilon$ (m is particle's mass), one obtains

$$\Omega_H \mathcal{J}_S < m\frac{k^2-1}{2} \tag{34}$$

It is very obvious from Eq. (34) that in Penrose process, the extraction energy is depending on the particle's mass. However, the ratio of black hole's angular momentum and its potential gravity are uncertain, even from its original paper [6]. But there is one thing for certain, *a massless particle cannot join this process*.

5. Extreme Conditions

For the next discussion, the case will be reduced on the black hole which a almost equal to M, where literally means: *black hole in its extreme point*. In this state, black hole's energy is same as its rotational energy. The major consequence of this condition leads black hole's surface gravity is nearly zero $\kappa \to 0$. Since the black hole got no surface gravity, it has no longer emits radiation. By conclusion from the previous discussion, there is no better way to steal black hole's mass via no-other than Hawking radiation. Due to the reason, the process for excessing a black hole into its extreme point is pointless. So basically, taking black hole's angular momentum for the first step is preferable.

There is a conclusion toward this case. It will be proved that there is impossible to reach extreme point $a = M$ by whatever methods which implied. In the beginning, one can define total energy of the hole by E_T and rotational energy by $E_{\mathcal{J}}$. By definition that rotational energy decreasing black hole's total energy, one can show

$$E_T - E_{\mathcal{J}} = E_0\,, \tag{37}$$

where E_0 can be regarded as black hole's net-energy. One can rearrange Eq. (37) as

$$E_T = E_{\mathcal{J}} + E_0\,, \tag{38}$$

which resemble the classical energy of $E = T + V$. It shows the resemblance of E_T with E, $E_{\mathcal{J}}$ with T, and E_0 with V. Since energy must be positive, it can be inferred that

$$E_T \geq E_{\mathcal{J}}\,, \tag{39}$$

which can be regarded as $E_{\mathcal{J}}$ is a subset of E_T ($E_{\mathcal{J}} \subset E_T$). It means $E_{\mathcal{J}}$ cannot exceed its total energy. Somehow, by implying that $E_{\mathcal{J}} = E_T$ is impossible to be reached, since it implied that $E_0 = 0$, which means there is no potential gravity, no mass indeed. In another side, if condition $E_{\mathcal{J}} > E_T$ does exist, then E_0 must has negative value, which is theoretically can be considered as the appearing of a white hole.

6. Another Speculations

From the previous section, it is impossible to reach black hole's extreme point. Since it is impossible to have rotational energy of the hole without mass. It means, extreme condition only happened when black hole no longer has its mass, impossible indeed. However, one can regard this result by noting that by spinning the hole will decrease the mass of the hole by enormous centrifugal force which can pull δM out the horizon. Since inside the event horizon there is no escape for particles, consequently this speculation has no strong basis of argument but worth mentioned. The reason mentioning this section due to the fact that [1]-[4] ignored this fact in order to reach their conclusions.

7. Conclusion

Based on discussions in the previous sections, technically, Hawking radiation plays the main role of the destroying process. To put it simply, one need to remove black hole's angular momentum and lets Hawking radiation plays its role. Since the theoretical explanation obtained by Penrose process won't help so much in decreasing black hole's angular momentum and mass, depending this effect into account won't help the destroying process greatly. In another case, putting a black hole to surpasses its rotational extremity doesn't seem to be realistic, since there is a term of negative energy included

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